THE CANTOR EXPANSION OF A NUMBER

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By comparison, think of the base 8 representation (for example) of an integer x. This consists of a sequence of digits $b_n, \ldots, b_0$ such that $0 \le b_i \le 7$ and $x = 8^n b_n + \cdots + 8^2 b_2 + 8b_1 + b_0$.

Analogously, the **cantor expansion** of x consists of coefficients $a_n, \ldots, a_1$ such that $0 \le a_i \le i$ and

$$x = a_n\, n! + \cdots + a_2\, 2! + a_1 1!\ .$$

The following algorithms find the cantor expansion of an integer x and, conversely, compute x from its cantor expansion.

```
procedure decimal-to-cantor(x: positive integer)
n := 1
y := x          { y is a temporary variable used so that
                this procedure won't destroy the original value of x.}
while y ≠ 0
begin
    a_n := y mod (n+1)
    y := (y-a_n)/(n+1)
    n := n + 1
end
        { The expansion for x will be a_n n! + a_{n-1}(n-1)! + ... + a_2 2! + a_1 .  }
```

```
procedure cantor-to-decimal( a_n , ... ,a_1 : integers with 0 ≤ a_i ≤ i )
x := 0
for i = n to 1
begin
    x := x + a_i
    x := i*x
end
        { x is the value of the number whose cantor expansion
    has the digits a_n , ... ,a_1 .  }
```