MATH 611 **Due Wed Oct 14, 1992**

1. Prove that a **necessary** condition that a submodule N of an R-module M be a direct summand of M is that M/N be isomorphic to a submodule of M.

2. Let R be an integral domain, let M be an R-module and N a submodule of M. Prove that M/N is torsion free if and only if $t(M) \subseteq N$. (Here $t(M)$ denotes the torsion submodule of M.)

3. Let $R = \mathbb{Z}$ and let p be a prime number. Prove that if n is not a multiple of p then the map $x \mapsto nx$ is an automorphism of $\mathbb{Z}(p)$.

4. Let $R = \mathbb{Z}$ and $M = \prod_p \mathbb{Z}(p)$, where the product is taken of the set of prime numbers. Recall that, by definition, $\bigoplus_p \mathbb{Z}(p)$ is a submodule of $\prod_p \mathbb{Z}(p)$.

a) Prove that $\bigoplus_p \mathbb{Z}(p) = t(M)$.

b) Prove that if $n \neq 0 \in R$ then the map $x \mapsto nx$ is an automorphism of $M/t(M)$.

c) Prove that if p is a prime number and $x \in M/t(M)$ then $x \in p(M/t(M))$.

(In other words, there exists $y \in M/t(M)$ such that $x = py$.)

d) Prove that if $x \neq 0 \in M$ then there exists a prime number p such that $x \notin pM$.

e) Prove that $t(M)$ is not a direct summand of M.