Some Equivalent Statements
about Vectors and Subspaces

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The sentences in each group below are all equivalent to each other. Furthermore, the sentences in the third group are all true.

✓ W is a subspace of V.
✓ If two vectors are in W then so is their sum; furthermore, any scalar multiple of a vector in W will still be in W.
✓ If $\mathbf{w}_1 \in W$ and $\mathbf{w}_2 \in W$ then $\mathbf{w}_1 + \mathbf{w}_2 \in W$; furthermore, $c\mathbf{w}_1 \in W$ for every scalar c.

✓ $\mathbf{x}_1$ is a solution to the system $A\mathbf{x} = \mathbf{b}$.
✓ $A\mathbf{x}_1 = \mathbf{b}$.

✓ The set of solutions to the homogeneous system $A\mathbf{x} = \mathbf{0}$ is a subspace.
✓ If $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions to this system, then so is $\mathbf{x}_1 + \mathbf{x}_2$; furthermore, so is $c\mathbf{x}_1$ for any real number c.
✓ If $A\mathbf{x}_1 = \mathbf{0}$ and $Ax_2 = \mathbf{0}$ then $A(\mathbf{x}_1 + \mathbf{x}_2) = \mathbf{0}$; furthermore, for every real number c, $A(c\mathbf{x}_1) = \mathbf{0}$.