SOME EXTRA MIN-MAX PROBLEMS
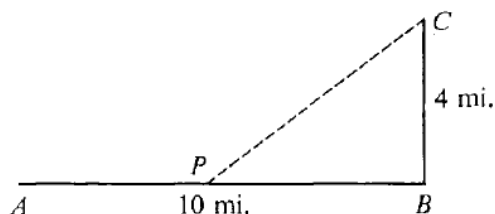
① Find the maximum value of x^3y if x and y belong to $[0, 1]$ and $x + y = 1$.

② A man can travel 5 mph along the path AB and 3 mph off the path as shown in the figure. Find the quickest route APC from the point A to the point C.


③ A professional football team has a stadium which seats 60,000. It is found that x tickets can be sold at a price of $p = 10 - x/10{,}000$ dollars per ticket. Find the values of x and p at which the total money received will be a maximum.

④ Find the real number which most exceeds its square.

⑤ Find the shortest distance between the curve $y = 2/x$ and the origin.

⑥ A warehouse is to be built in the shape of a rectangular solid with a square base. The cost of the roof per unit area is three times the cost of the walls. Find the shape which will enclose the maximum volume for a given cost.

⑦ The light intensity from a light source is equal to S/D^2 where S is the strength of the source and D the distance from the source. Two light sources A and B have strengths $S_A = 2$ and $S_B = 1$ and are located on the x-axis at $x_A = 0$ and $x_B = 10$. Find the point $x, 0 < x < 10$, where the total light intensity is a minimum.

⑧ Given $S > 0$. Prove that among all positive numbers x and y with $x + y = S$, the sum $x^2 + y^2$ is smallest when $x = y$.

⑨ A log 12 feet long has the shape of a frustum of a right circular cone with diameters 4 feet and $(4 + h)$ feet at its ends, where $h \geq 0$. Determine, as a function of h, the volume of the largest right circular cylinder that can be cut from the log, if its axis coincides with that of the log.

THE FOLLOWING PROBLEMS ARE FROM A VERY NICE 1916 TEXT BY H.B. PHILLIPS CALLED *DIFFERENTIAL CALCULUS.* THIS TEXT IS AVAILABLE FOR FREE AT www.archive.org

.17. The top, bottom, and lateral surface of a closed tin can are to be cut from rectangles of tin, the scraps being a total loss. Find the most economical proportions for a can of given capacity.

21. A circular filter paper of radius 6 inches is to be folded into a conical filter. Find the radius of the base of the filter if it has the maximum capacity.

My attempt at a picture for this one:
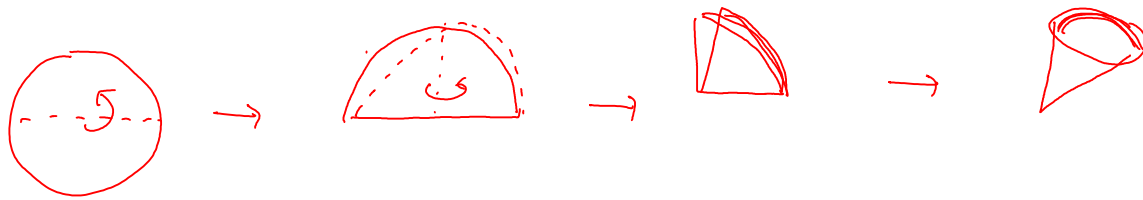
22. Assuming that the intensity of light is inversely proportional to the square of the distance from the source, find the point on the line joining two sources, one of which is twice as intense as the other, at which the illumination is a minimum.

29. A ship B is 75 miles due east of a ship A. If B sails west at 12 miles per hour and A south at 9 miles, find when the ships will be closest together.

30. A man on one side of a river $\frac{1}{2}$ mile wide wishes to reach a point on the opposite side 5 miles further along the bank. If he can walk 4 miles an hour and swim 2 miles an hour, find the route he should take to make the trip in the least time.