

Two class examples worked fully

1 Optimize $f(x) = x + 1/x^2$ on $[1, \sqrt{2}]$

$$f'(x) = 1 - 2x^{-3} \stackrel{\text{set}}{=} 0, \therefore 1 = 2x^{-3}, \therefore x^3 = 2, \therefore x = \sqrt[3]{2}$$

So:	x	$f(x)$
crit. pt	$\sqrt[3]{2}$	$\sqrt[3]{2} + 2^{-2/3} \approx 1.889$
endpoints	$\begin{cases} 1 \\ \sqrt{2} \end{cases}$	$\begin{cases} 1 + 1/1 = 2 \\ \sqrt{2} + 1/2 \approx 1.91 \end{cases}$

so f is maximized at $x=1$, minimized at $x=\sqrt[3]{2}$

(We'll see later a way to solve this without a calculator!)

2 Optimize $f(x) = \sin 2x - x$ on $[0, 4\pi]$

$$\begin{aligned} f'(x) &= 2\cos 2x - 1 \quad \text{set } = 0, \\ 2\cos 2x &= 1 \\ \therefore \cos 2x &= 1/2 \end{aligned}$$

This means that $2x = \pi/3, 5\pi/3, 7\pi/3, 11\pi/3, 13\pi/3, 17\pi/3, 19\pi/3, \dots$

(We only need to consider positive angles since $x \geq 0$)

$$\therefore x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6, 13\pi/6, 17\pi/6, 19\pi/6, \underbrace{25\pi/6, \dots}_{>4\pi, \text{ so ignore!}}$$

So:	x	$f(x)$
endpts	$\begin{cases} 0 \\ 4\pi \end{cases}$	$\begin{cases} 0 \\ -4\pi \end{cases}$
	$\pi/6$	$\sin \frac{\pi}{3} - \frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{\pi}{6} > 0$
	$5\pi/6$	$\sin \frac{5\pi}{3} - \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{5\pi}{6} < 0$
	$7\pi/6$	$\frac{\sqrt{3}}{2} - \frac{7\pi}{6}$
	$11\pi/6$	$-\frac{\sqrt{3}}{2} - \frac{11\pi}{6}$
	$13\pi/6$	$\frac{\sqrt{3}}{2} - \frac{13\pi}{6}$
	$17\pi/6$	$-\frac{\sqrt{3}}{2} - \frac{17\pi}{6}$
	$19\pi/6$	$\frac{\sqrt{3}}{2} - \frac{19\pi}{6}$

so f has a min of 4π at $x=4\pi$, max of $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$ at $x=\pi/6$

Note I did not need to evaluate anything, I know that π is just over 3 and $\sqrt{3} \approx \frac{\text{George Washington's Birthday}}{1000} = 1.732$, so $\frac{\sqrt{3}}{2} > .875 > \frac{\pi}{6}$!