

Derivatives

Theorem 0.1. (*Derivatives of basic functions*)

1. If $f(x) = k$ where k is a constant, then $f'(x) = 0$

$$\frac{d}{dx}k = 0$$

2. If $f(x) = x^n$ where $n > 0$ is a positive integer, then

$$f'(x) = nx^{n-1}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

3. If $f(x) = \sqrt{x}$ and $x > 0$ then $f'(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-1/2}$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

4. If $f(x) = \sin x$ then $f'(x) = \cos x$

$$\frac{d}{dx}\sin x = \cos x$$

5. If $f(x) = \cos x$ then $f'(x) = -\sin x$

$$\frac{d}{dx}\cos x = -\sin x$$

Proof. Class. □

Theorem 0.2. (*Construction rules*)

Assume f and g are differentiable at x , and k is a constant.

$$1. (kf)'(x) = kf'(x)$$

$$\frac{d}{dx} kf(x) = k \frac{d}{dx} f(x)$$

$$2. (f + g)'(x) = f'(x) + g'(x)$$

$$\frac{d}{dx} (f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$$

$$3. (f - g)'(x) = f'(x) - g'(x)$$

$$\frac{d}{dx} (f(x) - g(x)) = \frac{d}{dx} f(x) - \frac{d}{dx} g(x)$$

$$4. (fg)'(x) = f(x)g'(x) + f'(x)g(x)$$

$$\frac{d}{dx} (f(x)g(x)) = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$$

$$5. \left(\frac{f}{g}\right)'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{g(x)^2}$$

Proof. Class. □

Example 0.1. (*Applications: derivatives of some more functions*)

1. Polynomials: $\frac{d}{dx}(a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_nx^n) = ?$

2. $\frac{d}{dx}x^n = ?$ when $n < 0$ is an integer and $x \neq 0$

3. Other trig functions: $\frac{d}{dx} \tan x = ?$

$\frac{d}{dx} \sec x = ?$

$\frac{d}{dx} \cot x = ?$