

**Math 241, S17**  
**Quiz 7**Name: *Soln*  
Section:

INSTRUCTIONS: Write legibly. Show all work; explain your answers.

- (10) 1. If
- $f(\theta) = 3\theta^2 - 4\theta^3$
- , find the intervals on which the function is increasing and decreasing, and all local and absolute extreme values.

$$f'(\theta) = 6\theta - 12\theta^2 = 6\theta(1-2\theta) = 0, \quad \theta = 0 \text{ or } \theta = 1/2$$

$\uparrow$   
 set

$f' \quad - - - 0 + + + 0 - - -$   
 $\quad \quad \quad 0 \quad \quad \quad 1/2$

so  $f$  is increasing on  $[0, 1/2]$   
 decreasing on  $(-\infty, 0], [1/2, \infty)$

Local min of  $f(0) = 0$  at  $x = 0$   
 Local max of  $f(1/2) = 3/4 - 4/8$  at  $x = 1/2$  } By 1<sup>st</sup> derivative test

Since  $f(\theta) = \theta^3(3\theta - 4) \rightarrow \pm\infty$  as  $\theta \rightarrow \pm\infty$ ,

There is no global min or max

- (10) 2. Suppose that the derivative of a function
- $y = f(x)$
- is given by the formula

$$y' = (x-1)^2(x-2)$$

At what points does the graph of the function have a local minimum, local maximum, or point of inflection? (Hint: draw the sign pattern for  $y'$  and  $y''$ .)

$$y'' = (x-1)^2 \cdot 1 + 2(x-1)(x-2) = (x-1)[x-1+2x-4] = (x-1)(3x-5)$$

$$y' = 0 \Leftrightarrow x = 1 \text{ or } 2$$

$$y'' = 0 \Leftrightarrow x = 1 \text{ or } 5/3$$

$y'' \quad + + 0 - - - 0 + + + + + + + + +$   
 $y' \quad - - 0 - - - - - 0 + + + + + + + + +$   
 $\quad \quad \quad 1 \quad \quad \quad 5/3 \quad \quad \quad 2$

local min at  $x = 2$  (1<sup>st</sup> deriv test or 2<sup>nd</sup> deriv. test)

No local max

points of inflection at  $x = 1, x = 5/3$  (concavity changes)

- (20) 3.  $f(x) = \frac{x^2-3}{x-2}$ ,  $c \neq 2$ , find the intervals on which the function is increasing and decreasing, the intervals on which it is concave up or down, and all local and absolute extreme values and inflection points. Then sketch the curve.

$$f'(x) = \frac{(x-2)(2x) - (x^2-3)(1)}{(x-2)^2} = \frac{2x^2-4x-x^2+3}{(x-2)^2} = \frac{x^2-4x+3}{(x-2)^2} = \frac{(x-1)(x-3)}{(x-2)^2}$$

$$f''(x) = \frac{(x-2)^2(2x-4) - (x^2-4x+3)(2)(x-2)}{(x-2)^4} = \frac{2(2x+1)}{(x-2)^3}$$

$$f' = 0 \Leftrightarrow x = 1 \text{ or } 3$$

$$f'' = 0 \Leftrightarrow x = -1/2$$

$f'$	+	+	0	-	-	-	-	0	+	+	+	+	+	+	+
$f''$	+	+	+	+	+	+	0	-	-	-	-	0	+	+	+
	$-1/2$						1		2			3			

Increasing on  $(-\infty, 1]$ ,  $[3, \infty)$  Decreasing on  $[1, 2)$ ,  $(2, 3]$

Concave up on  $(-\infty, -1/2)$ ,  $(2, \infty)$  down on  $(-1/2, 2)$

local max at  $x=1$ , local min at  $x=3$ , inflection point at  $x=-1/2$

Since  $f(x) \rightarrow \pm\infty$  as  $x \rightarrow \pm\infty$ , no global min or max

Note vertical asymptote at  $x=2$ .

