To show: $\langle x,y\rangle = \langle z,w\rangle \iff x=z \text{ \& } y=w$

The direction $\Leftarrow$ is easy, we did it in class.

$\Rightarrow$) Assume $\langle x,y\rangle = \langle z,w\rangle$, That is, $\{\{x\},\{x,y\}\} = \{\{z\},\{z,w\}\}$

Case 1 $\{x\} = \{x,y\}$. Then by extension $\{x\} = \{x,y\} = \{z\} = \{z,w\}$,

so (by extension again) $x = y = z = w$.

Case 2 $\{x\} \neq \{x,y\}$. Note (by extension) $\{z\} \neq \{z,w\}$, so $x \neq y$, $z \neq w$.

By extension either $\{x\} = \{z\}$ or $\{x\} = \{z,w\}$. However, if

$\{x\} = \{z,w\}$ then $z = x = w$, but since $z \neq w$ we must have $\{x\} = \{z\}$,

so $x = z$. By extension $\{x,y\} = \{z,w\}$, and since $y \neq x = z$, $y = w$.

In either case, $x = z$ and $y = w$.