M632 Non-text problems on uniform stuff

1. Let f_n be a sequence of continuous functions from a metric space X to a metric space Y which converges to a function f uniformly on compact sets. Show that f is continuous. (Warning: closed balls in metric spaces need not be compact!)

2. For $1 \leq p < \infty$, let (as usual) ℓ^p be the space of all sequences x_n such that $\sum_{n=1}^{\infty} |x_n^p| < \infty$ with the $\|\|_p$ norm. Show that the closed unit ball of ℓ^p is not compact.

3. Let λ be Lebesgue measure on the line, Let $g \in L^1([0,1], \lambda)$, and let $\mathcal{F}$ be the set of all absolutely continuous functions f on $[0,1]$ such that $f(0) = 0$ and $|f'| < |g|$ almost everywhere. Show that the closure of $\mathcal{F}$ is compact (in the uniform metric on $C([0,1])$).