Math 671 - Assignment 4 - Due September 27

1. Prove that if $X \sim \mathcal{N}(0,1)$ then $X^2 \sim \Gamma(1/2, 2)$

   (Note: We say
   $$Y \sim \Gamma(\alpha, \beta), \qquad (\alpha, \beta > 0)$$
   provided Y has pdf
   $$f_{\alpha,\beta}(t) = \frac{1}{\Gamma(\alpha)\beta^\alpha} t^{\alpha-1} e^{-t/\beta} I_{t>0}$$
   where $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ is the gamma function. You should be able to do this without actually knowing what $\Gamma(1/2)$ is, in fact this should help you compute it probabilistically!)

2. (Don't turn in, but - really! - do this) Let $(\Omega, \mathcal{A}, P)$ be a probability space. If $\mathcal{I}_1, \ldots, \mathcal{I}_n \subseteq \mathcal{A}$ are independent π-systems with $\Omega \in \mathcal{I}_j$ for all j then $\sigma(\mathcal{I}_1), \ldots, \sigma(\mathcal{I}_n)$ are independent.

3. Text E4.3

4. Text 4.12 (p48)