2. A box contains three red and five blue balls. Define a sample space for the experiment of recording the colors of three balls that are drawn from the box, one by one, with replacement.

Let \( \Omega = \{ (x_1, x_2, x_3) : x_i \in \{R, B\}, i = 1, 2, 3 \} \). In this representation, \( x_i = R \) if a red ball is selected on the \( i \)-th draw and \( x_i = B \) if a blue ball is selected on the \( i \)-th draw.

6. Define a sample space for the experiment of drawing two coins from a purse that contains two quarters, three nickels, one dime, and four pennies. For the same experiment describe the following events:

(a) drawing 26 cents;

(b) drawing more than 9 but less than 25 cents;

(c) drawing 29 cents.

\[ \Omega = \{ qq, qn, qd, qp, nn, nd, np, dp, pp \} \]. In this representation, \( q \) = quarter, \( n \) = nickel, \( d \) = dime, \( p \) = penny. Thus, \( qq \) means two quarters were drawn and \( np \) means one nickel and one penny were drawn.

(a) 26 cents = \{qp\}.

(b) more than 9 but less than 25 cents = \{nn, nd, dp\}.

(c) drawing 29 cents = \( \emptyset \).

Remark: Other sample spaces are possible. For instance, you could take

\[ \Omega = \{(x, y) : x, y \in \{q_1, q_2, n_1, n_2, n_3, d, p_1, p_2, p_3, p_4\} \text{ and } x \neq y \} \].
18. Define a sample space for the experiment of putting in a random order seven
different books on a shelf. If three of these seven books are a three volume
dictionary, describe the event that these volumes stand in increasing order side by
side (i.e., volume I precedes volume II precedes volume III).

Assign one of the letters \( A, B, C, D, E, F, G \) to each
book so that each letter is used exactly one time.
Let \( \Omega \) be the set of all seven letter words made
from the letters \( A, B, C, D, E, F, G \) such that no
letter is repeated. Then each word represents
one and only one arrangement of books on the
shelf.

Now, suppose that books \( A, B, \) and \( C \) represent volumes
I, II, and III, respectively, of the dictionary. View the
string \( ABC \) as a single letter. Then, the set of all
words that we can make from the five letters
\( ABC, D, E, F, G \) such that no letter is repeated and all
the letters are used is the set of words that represent
the event of volumes I, II, and III standing side by side
and in order.

Remark: Another option for the sample space is the
set of all one-to-one functions

\[
f : \{A, B, C, D, E, F, G\} \rightarrow \{1, 2, 3, 4, 5, 6, 7\}
\]

where \( f(A) \) is the position of book \( A \), and so on. Having
the three volumes side by side and in order would mean
\( f(B) = f(A) + 1 \) and \( f(C) = f(B) + 1 \).
19. Let \( \{ A_1, A_2, A_3, \ldots \} \) be a sequence of events. Find an expression for the event that infinitely many of the \( A_i \)'s occur.

Let \( \Omega \) be the sample space and let \( x \in \Omega \). Then \( x \) is an element of infinitely many of the \( A_i \)'s if and only if for every natural number \( N \) there exists a natural number \( n \geq N \) such that \( x \in A_n \). Defining the sets

\[
B_N = \bigcup_{i=N}^{\infty} A_i,
\]

we can restate the condition as: for every natural number \( N \), \( x \in B_N \). Therefore, \( x \) occurs in infinitely many of the \( A_i \)'s if and only if \( x \in B_N \) for all natural numbers \( N \). This is equivalent to the condition

\[
x \in \bigcap_{N=1}^{\infty} B_N = \bigcap_{N=1}^{\infty} \bigcup_{i=N}^{\infty} A_i.
\]

We conclude that \( \bigcap_{N=1}^{\infty} \bigcup_{i=1}^{\infty} A_i \) represents the event that infinitely many of the events \( A_1, A_2, A_3, \ldots \) occur.