MATH 475 FINAL EXAM Friday May 13, 1994

Do **seven** problems.

(65) **1.** How many ways are there of

a) choosing n items from the multiset $\{\infty \cdot a_1, \infty \cdot a_2, \ldots, \infty \cdot a_r\}$? (Order is not important here, only the set of elements selected.)

b) permuting the multiset $\{3 \cdot a_1, 4 \cdot a_2, 5 \cdot a_3\}$?

c) placing 3 red rooks, 4 blue rooks, and 1 yellow rook on a 9×15 chessboard so that no rook attacks any other?

d) seating 10 men, 7 women, and a koala bear around a circular table so that no woman is seated next to another woman?

e) solving the equation $w + x + y + z = 30$ if w, x, y and z are required to be integers with $5 \geq w \geq 0$, $x \geq 2$, $y \geq 3$, $z \geq 6$?

(15) **2.** Let h_n be the number of ways of selecting n items from the multiset $\{3 \cdot e_1, \infty \cdot e_2, \infty \cdot e_3, \infty \cdot e_4\}$ such that there are an odd number of copies of e_2 and the number of copies of e_4 is a multiple of 5.

Find the generating function for the sequence h_n.

(15) **3.** Find the generating function for the recurrence relation

$$h_n = 2h_{n-1} + 3n, \qquad h_0 = 4.$$

(40) **4.** Solve the recurrence relations

a) $h_n = 2h_{n-1} + 2h_{n-2}$, $h_0 = 0$, $h_1 = -1$.

b) $h_n = 3h_{n-1} + 2 \cdot 3^n$, $h_0 = 5$.

(20) **5.** A chess master has decided to play 20 games during a period of 15 days, playing at least one game every day. Prove the there will be a consecutive sequence of days during which he plays exactly 9 games.

(20) **6.** Prove that the sum of the degrees of the vertices in any graph equals twice the number of edges.

(25) **7.** Give the details of the following proof that the Ramsey number $r(3,3)$ is 6, i. e. if all possible edges are drawn between at least six vertices and each edge is colored either red or blue, then there will be at least one monochromatic triangle.

a) Prove that if there are at least 6 vertices then it is possible to find 4 vertices O, P, Q, and R such that the edges OP, OQ, and OR all have the same color.

b) Prove that if O, P, Q, and R are as above, then some three of these four vertices will determine a monochromatic triangle.

(30) **8.** Recall that in a bipartite graph, a **matching** M is a set of edges of the graph with no vertices in common and a **cover** $\mathcal{C}$ is a set of vertices such that at least one vertex of every edge is in $\mathcal{C}$.

a) Prove that there are always at least as many vertices in any cover as there are edges in any matching.

b) Prove that if a matching M has the same number of edges as the number of vertices in some cover $\mathcal{C}$, then M is a maximal matching.