What is symmetry?

The simplest type is “mirror symmetry”, or “reflection symmetry”.

The figure on the left (a triangle) is symmetric about the green vertical line. The middle figure (a rectangle) is symmetric about two green lines, one vertical and one horizontal. The right figure is symmetric about the horizontal line, but not about any vertical line.

A reflection is a way of moving a figure. You have a line, called the axis of the reflection, of points that don’t move at all (such points are called fixed points of the motion). Every other point moves to the point directly opposite to it across this line.

The reflection about the vertical line takes A to A* and B to B*. 
A reflection symmetry occurs when there is a reflection which, when applied to a figure, moves that figure identically on top of itself. Reflect the triangle ABC about the green line and you get the line AB*C*, which is a different triangle, so this is not a symmetry:

A reflection is a type of **rigid motion**, which means a motion which doesn’t change distances or angles: if points A and B are moved to points A* and B*, then the distance from A to B is the same as the distance from A* to B*.

In general, a **symmetry** of a figure is any rigid motion which moves the figure exactly on top of itself.

There are four types of rigid motions in the plane: reflections, rotations, translations and glide reflections. Reflections and rotations are the only ones having fixed points; they are the only types of symmetries that finite figures can have. We will discuss them today. We’ll study the others, and their symmetries, next time.
**Rotations.** A rotation has exactly one fixed point (unlike reflections, which have a whole line of fixed points); this fixed point is called the **center of the rotation**, or **rotocenter**. The rotation moves each line through the center to another line through the center, and each line is moved by the same angle. A 90° clockwise rotation with rotocenter A moves line AB to AC, and moves AC to AD.

![Diagram of rotation](https://example.com/diagram.png)

A figure has **rotation symmetry** if there is a rotation that moves the figure on top of itself.

Look again at these three figures:

![Triangle and rectangle](https://example.com/triangle.png)

The left and right figures have no rotation symmetry. The rectangle in the middle does have a rotation symmetry. The rotocenter is the point right in the middle of the rectangle; rotate the rectangle 180° (either clockwise or counter-clockwise, it makes no difference). The point A moves to C and B moves to D.

The left and right figures have one reflection symmetry, and no other symmetries. So they have **the same kind of symmetry**. The rectangle in the middle is **more symmetrical**; it has two different reflection symmetries, and one rotation symmetry.
There is another rigid motion called the identity; it is the “motion” in which every point stays where it is. Since nothing moves, you could object to calling this a motion, but the book includes it, so I will too. The identity can be considered to be a 0° rotation. It is a symmetry of every figure. So including the identity, the rectangle had four symmetries: two reflections and two rotations.

Some figures have only rotations for symmetries. We say a figure has symmetry of type $Z_n$ if it has exactly $n$ rotation symmetries (one of which is the identity).

Examples of $Z_n$ symmetry:

\[
\begin{align*}
Z_1 & \\
Z_2 & \\
Z_3 & \\
Z_4 & 
\end{align*}
\]

Some figures have reflections and rotations for symmetries. It’s a mathematical theorem that the numbers of reflection and rotation symmetries have to be the same. We say a figure has symmetry of type $D_n$ if it has exactly $n$ reflection symmetries and $n$ rotation symmetries (one of which is the identity).

Examples of $D_n$ symmetry:

\[
\begin{align*}
D_1 & \\
D_2 & \\
D_3 & \\
D_4 & 
\end{align*}
\]

isosceles triangle  rectangle  equilateral triangle  square

The $Z_n$ and $D_n$ symmetry types are the only symmetry types a finite figure can have.