Section 5. The last-diminisher method.

- \( N \) represents the number of players (at least 3).

- First order the players: \( P_1, P_2, P_3 \) etc.

- Basic principle: the first player in each round marks a piece and claims it. The part claimed is called \( C \) and the remainder is called \( R \). The next player can pass; if he doesn’t pass, he must diminish the size of \( C \) and claim this smaller piece for himself. This continues with each successive player. When a player is claiming a piece, he doesn’t know if he will get that piece, or a later player will, so he must claim a piece which is a fair share for him (but no larger – actually the last player in the round can claim a larger than fair share, since no one after him can take it away).

- Round 1. Player \( P_1 \) marks a piece worth \( 1/N \) of the whole (in \( P_1 \)’s valuation of worth). \( P_1 \) is the claimant of that piece. If that piece is worth \textbf{less than or equal to} \( 1/N \) of the whole to \( P_2 \), then \( P_2 \) passes; if it is worth more than \( 1/N \) of the whole to \( P_2 \), then \( P_2 \) makes the piece smaller, so it is worth \( 1/N \) of the whole (in \( P_2 \)’s valuation), and \( P_2 \) becomes the claimant of the new smaller piece. The rest of the players do the same, either passing or claiming a part of the piece which is worth \( 1/N \) of the whole to them. The last player to claim the piece gets it, and is finished.
Here is an example from the textbook: \( N=5 \)

**Move 1**
(by \( P_1 \))
CLAIM

**Current Status**
Claimant: \( P_1 \)
Nonclaimants: \( P_2, P_3, P_4, P_5 \)

Comments: \( P_1 \) considers \( C \) to be worth 20\% and \( R \) to be worth 80\% of the total value of the island.

**Move 2**
(by \( P_2 \))
PASS

**Current Status**
Claimant: \( P_1 \)
Nonclaimants: \( P_3, P_4, P_5, P_2 \)

Comments: \( P_2 \) passes (he considers \( C \) to be worth less than or equal to 20\% of the total value of the island).
**Move 3**  
(by $P_3$)  
**DIMINISH**  

**Current Status**  
Claimant: $P_3$  
Nonclaimants: $P_4, P_5, P_2, P_1$

**Comments:** $P_3$ considers $P_1$'s claim to be worth *more* than 20% of the total. $P_3$ diminishes it to a new $C$ worth exactly 20% of the total and becomes its claimant. $P_1$ becomes a nonclaimant in contention for a fair share of the new $R$.

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**Move 4**  
(by $P_4$)  
**DIMINISH**

**Current Status**  
Claimant: $P_4$  
Nonclaimants: $P_2, P_1, P_3, P_5$

**Comments:** $P_4$ considers $C$ to be worth *more* than 20% of the total. $P_4$ diminishes it to a new $C$ worth 20% of the total and becomes its claimant. $P_1$ becomes a nonclaimant in contention for a fair share of the new $R$.

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**Move 5**  
(by $P_5$)  
**PASS**

**Current Status**  
Claimant: $P_4$  
Nonclaimants: $P_2, P_1, P_3, P_5$

**Comments:** $P_5$ considers $C$ to be worth *less* than 20% of the total value of the island and passes. All players have now had a chance to diminish or pass. Round 1 is over, with $C$ going to the last diminisher ($P_4$).
• Round 2. Now there are N-1 players dividing up the remainder of the cake. They do the same procedure as in round 1, except they pass if the piece is worth no more than 1/(N-1) of the whole for this round. In the books example, N-1=4.
Round 3 of books example: N-2=3 players left:

- Since one player drops out after each round, after N-2 rounds there are only 2 players left.
• Round N-1. There are only two players left. They split the remainder of the original property between themselves by the divider-chooser method (as discussed in section 2).
Another example. Four players divide a cake by the last diminisher method. The players in order are P1, P2, P2 and P4.

How many rounds are there?

Answer: 3, one fewer than the number of players.

Suppose P2 and P3 are the only diminishers in round 1 and there are no diminishers in round 2.

Which player gets her fair share at the end of round 1?

Answer: P3 is the last diminisher, hence the last claimant, hence gets the piece she claimed.

Which player gets her fair share at the end of round 2?

Answer: P1 is the last diminisher, hence the last claimant, hence gets the piece she claimed.

Which player is the first to cut the cake at the beginning of round 3?

Answer: P1 already got her piece, so is finished. P2 is the next player, so cuts the cake first in this round.

Then what happens?

Answer: P4 claims one of the two pieces that P2 cut the cake into. P2 takes the piece left over.
Section 6. The method of sealed bids.

This is a method for dividing goods which can’t be subdivided into arbitrarily small pieces; things like houses, cars, paintings, etc.

Example. Three players (A, B and C) which to divide up three items.

- **Step 1 (The bids).** Each player makes a sealed bid stating the monetary value of each item to him or her. The bids are opened, revealing:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item 1</td>
<td>$36</td>
<td>$45</td>
<td>$39</td>
</tr>
<tr>
<td>Item 2</td>
<td>$15</td>
<td>$18</td>
<td>$9</td>
</tr>
<tr>
<td>Item 3</td>
<td>$21</td>
<td>$18</td>
<td>$24</td>
</tr>
</tbody>
</table>

- **Step 2 (The allocations).** Each item goes to the highest bidder. So B gets items 1 and 2, C gets item 3, and A gets nothing.
• **Step 3 (The payments).** Calculate what each player considers the totality of the items to be worth, and divide by 3 to get that players **fair share** of the total.

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<td>$18</td>
<td>$24</td>
</tr>
<tr>
<td>Total</td>
<td>$72</td>
<td>$81</td>
<td>$72</td>
</tr>
<tr>
<td>Fair share</td>
<td>$24</td>
<td>$27</td>
<td>$24</td>
</tr>
</tbody>
</table>

Player B received items 1 and 2, which B values at $45+$18=$63, but B’s fair share is only $27, so B has to pay $63-$27=$36 to the Estate.

Player C received item 3 worth $24 (to C), which is exactly C’s fair share, so C is done.

Player A received no items, so the Estate gives A $24, so that A also receives his fair share.

All three players have received, either in items or in money, what they judge to be their fair share.

• **Step 4 (Dividing the surplus).** The estate received $36 from B and paid $24 to A, so has a surplus of $12. This is split in three equal portions $4 each, and one portion given to each player.

So each player receives **more** than what that player considered a fair share.