Chapter 5. Euler Circuits

Section 1. Routing Problems.

Suppose we have some destinations, and some road or paths connecting them. We want to find a route (that is a choice of the roads/paths) which passes over every road (or path).

Some problems:
1. Is it possible to pass over each road exactly once?
2. If not, how can we pass over all the roads making as few as possible extra passes over roads?

Look at the examples in the book and on the blackboard.

We will simplify the pictures representing these examples by using what are called graphs.

Section 2. Graphs.

A graph is a picture consisting of dots, called vertices, and lines, called edges. The edges do not have to be straight, and can cross each other if necessary, but edges must connect two vertices. An edge is called a loop if it connects a vertex with itself.

We will usually label vertices with capital letters, and sometimes we will label edges with lower case letters.

Examples:
This graph has vertices A, B, C, D, and five edges AB, AC, AD, BC and BD. Note that there is no edge joining C to D. If we wanted an edge from C to D, we could draw it as either of the following:

Here is the first example with a loop AA added:
Section 3. Graph Concepts and Terminology

Adjacent vertices: two vertices with an edge joining them.

Adjacent edges: two edges meeting at a vertex.

Degree of a vertex: the number of edges coming from that vertex (loop counts as 2 at its vertex).

Path: a sequence of vertices, each adjacent to the next, with different starting and ending vertices. A path can pass through a vertex more than once, but can only pass through an edge once. In our first example, A, D, B, C is a path, and A, B, C is a path, but A, C, D is not a path, because no edge joins C to D. A, B, C, A is not a path because it starts and ends at the same vertex. A, B, C, A, B is not a path, because the edge AB is crossed twice.

Circuit: same as a path, except it starts and ends at the same vertex. A, B, C, A is a circuit, but A, D, B, C is not a circuit.

Connected graph: graph is connected if you can get from any vertex to any other by some path. The example above is connected, but the one below is not (it’s disconnected).

The part of the graph involving A, B, C and D is one component of this graph, the part involving E, F and G is the other component.
**Bridge**: An edge is called a bridge if, when you remove it from a graph, the graph changes from connected to disconnected. In the graph below, the edge BD and FG are bridges.

![Graph with edges BD and FG as bridges](image)

**Euler path**: a path that travels through every edge of a connected graph; so it travels through every edge **once and only once**.

**Euler circuit**: a circuit that travels through every edge of a connected graph; so it travels through every edge **once and only once**.

**Section 4. Graph models.**

Look at the examples in the book and the blackboard.

Here is a graph representing the Königsberg Bridge Problem:

![Graph representing the Königsberg Bridge Problem](image)