Review for Exam 4

Chapter 10

1. Percentage

.035% = .035/100 = .00035

Increase $1000 by 4.7%:
1000 + .047 x 1000 = 1.047 x 1000 = $1047

Decrease $200 by 20% : 200 - .2 x 200 = 0.8 x 200 = $160

2. Simple Interest.

Suppose $8000 earns 16% APR simple interest.

Then its future value after t years is:

\[ F(t) = 8000 \times (1 + .16 t) = 8000 + 8000 \times .16 t = 8000 + 1280 t \]

\[ F(1) = 9280, F(2) = 10,560, F(3) = 11,840 \text{ etc.} \]

$10,000 collects simple interest each year for 5 years. At the end of that time, a total of $12,000 is paid back. Determine the APR for this loan:

interest =$12,000 - $10,000 = $2000

interest per year = $2,000/5 = $400 per year.

APR = $400/$10,000 = .04 = 4%. 
3. Compound Interest.

If the APR is r, then the periodic interest rate is:

\[ p = r \] (yearly compounding)
\[ p = r/12 \] (monthly compounding)
\[ p = r/365 \] (daily compounding)

\[ \frac{F(t+1)}{F(t)} = 1 + p \] is called the common ratio.

Suppose $650 earns 6% APR interest compounded monthly. Then the future after 5 years is:

\[ $650 \times (1.005)^{60} \] (since \(.06/12 = .005\) and \(5\times12 = 60\) months)

4. Geometric sequences

\[ P + cP + c^2P + \cdots + c^{N-1}P = P\left(\frac{c^N - 1}{c - 1}\right) \]

5. Fixed deferred annuity: equal payments made or received over regular time intervals, so as to produce a lump-sum payment in the future.
Starting at age 25, Markus invests $2000 at the *end* of each year in an IRA with an APR of 7.5% compounded annually. How much money will there be in Markus's retirement account when he retires at the age of 65? (Assume that his 40th and last deposit generates *no* interest).

Total received at age 65 is

\[
2000 + 2000 \times 1.075 + \ldots + 2000 \times 1.075^{39} =
\]

\[
2000 (1 + 1.075 + \ldots + 1.075^{39}) =
\]

\[
$2000 (1.075^{40} - 1)/.075 = $454,513.04
\]

6. **Installment loans:** equal payments made at equal time intervals for the purpose of paying off a lump sum of money received up front.

Find the present value of an installment loan consisting of 25 annual payments of $3000 assuming an APR of 6% compounded annually. (Assume that the payments are made at the end of each year).

Solution. \( p = .06. \) The present values of the payments are:

1st: \( 3000/1.06, \) 2nd: \( 3000/1.06^2, \ldots \) 25th: \( 3000/1.06^{25} \)

So the total present value of all the payments is

\[
\frac{3000}{1.06} + \frac{3000}{1.06^2} + \ldots + \frac{3000}{1.06^{25}} =
\]

\[
\frac{3000}{1.06} (1 + \frac{1}{1.06} + \ldots + \frac{1}{1.06^{24}}) =
\]
\[ \frac{3000}{1.06} \left( \frac{1}{1.06^{25}} - 1 \right) / \left( \frac{1}{1.06} - 1 \right) = 38,350.07 \]
Chapter 11. Symmetry

**rigid motion** = motion that doesn’t change distances or angles. 4 types: reflection, rotation, translation, glide reflection

A **symmetry** of a figure is any rigid motion which moves the figure exactly on top of itself.

Reflection about green line:

\[ \begin{array}{c}
A \\
C \\
B
\end{array} \quad \leftrightarrow \quad \begin{array}{c}
A^* \\
B^* \\
C^*
\end{array} \]

90° clockwise rotation with center A:

\[ \begin{array}{c}
B \\
C \\
A \\
A^* \\
B^* \\
C^*
\end{array} \]

**Symmetry types of finite shapes.**

**Dn (n = 1, 2, 3, ...):** This is the symmetry type of an object with n reflection symmetries and n rotation symmetries.

**Zn (n = 1, 2, 3, ...):** This is the symmetry type of an object with n rotation symmetries but no reflection symmetries.

Examples of Zn symmetry:
Examples of $D_n$ symmetry:

- $D_1$: isosceles triangle
- $D_2$: rectangle
- $D_3$: equilateral triangle
- $D_4$: square

**translation:**

A $D_1$ isosceles triangle is translated to its image $C*$. The vertices $A$, $B$, and $C$ are mapped to $A*$, $B*$, and $C*$ respectively.

**glide reflection** by vector $v$ about the green line:

A $D_2$ rectangle is glide reflected along the green line to its image $C*$. The vertices $A$, $B$, and $C$ are mapped to $A*$, $B*$, and $C*$ respectively. The green line acts as the axis of reflection.

The axis is indicated by the blue segment between $A'$ and $B'$. The glide reflection is performed along this axis, with $v$ being the vector of translation.
What symmetries do the following border patterns have?

- ... JJJJJJJJJ ... only translations
- ... TTTTTTT ... translations and vertical reflections
- ... CCCCCC ... translations and horizontal reflections
- ... NNNNNN ... translations and half-turns
- ... qdqdqdqd ... translations and glide reflections
- ... qpdbqpdb ... transl., vert. refl., and glide reflections