SAMPLE EXAM 2 Answers Math 215

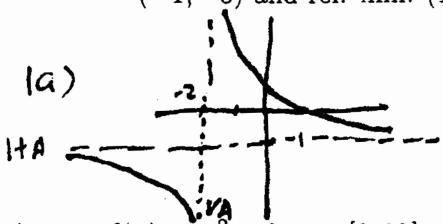
() **1.**

a. $f(x) = \frac{1-x}{x-2}$, $f'(x) = \frac{-3}{(x+2)^2}$, $f''(x) = \frac{6}{(x+2)^3}$: decr. on $(-\infty, -2)$ and $(-2, \infty)$; CD on $(-\infty, -2)$ and CU on $(-2, \infty)$. There are no extrema and no IPs.

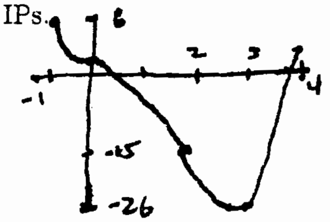
b. $f(x) = x^4 - 4x^3 + 1$ on $[-1, 4]$. $f'(x) = 4x^2(x-3)$, $f''(x) = 12x(x-2)$; decr. on $(-1, 0)$ and $(0, 3)$, incr. on $(3, 4)$; CU on $(-1, 0)$ and $(2, 4)$, CD on $(0, 2)$. Abs max $(-1, 6)$, rel. max. $(4, 1)$, abs. min. $(3, -26)$; IPs $(0, 1)$ and $(2, -15)$.

c. $f(x) = 2 + 2x - 3x^{2/3}$ on $[-1, 2]$. $f'(x) = 2 - 2x^{-1/3}$, $f''(x) = \frac{2}{3}x^{-4/3}$; incr. on $(-1, 0)$ and $(1, 2)$, decr. on $(0, 1)$; CU on $(-1, 0)$ and $(0, 2)$. Abs max $(0, 2)$, rel. max. $(2, 1.24)$, abs. min. $(-1, -3)$ and rel. min. $(1, 1)$; no IPs.

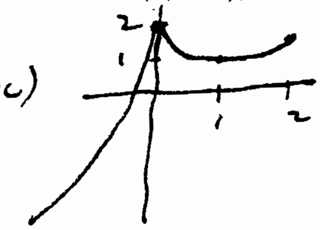
1a)



1b)



1c)



() **2.** $f(x) = x^2 - kx$ on $[0, 2k]$. $f'(x) = 2x - k = 0$ at $x = k/2$. $f(0) = 0$, $f(k/2) = -k^2/4$ (smallest value), $f(2k) = 2k^2$ (largest value).

() **3.** The line shown is $y = -\frac{2}{5}x + 2$. We want to maximize $A = xy = x(-\frac{2}{5}x + 2) = -\frac{2}{5}x^2 + 2x$. $0 \le x \le 5$. $dA/dx = -\frac{4}{5}x + 2 = 0$ at $x = \frac{5}{2}$. $A(0) = A(5) = 0$, so $A(\frac{5}{2}) = \frac{5}{2}$ is the max area (at $(\frac{5}{2}, 1)$).

() **4.** Suppose $x^3 + xy + y^2 = 7$.

a. $\frac{d}{dx}(x^3 + xy + y^2) = 3x^2 + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$. Solve to get $\frac{dy}{dx} = -(3x^2 + y)/(x + 2y) = -5/5 = -1$ at $x = 1$, $y = 2$.

b. $\frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = -1 \cdot 3 = -3$ at $x = 1$, $y = 2$.

() **5.** $V = \pi r^2 h$. $0 = \frac{dV}{dr} = \pi(2rh + r^2\frac{dh}{dr})$ so $\frac{dh}{dr} = -2h/r = -2$ when $h = 2$ and $r = 2$.

() **6.** We partition [0,4] into four intervals of length $\Delta x = 1$: [0,1], [1,2], [2,3], [3,4], with midpoints: .5, 1.5, 2.5, 3.5. The midpoint method approximates the integral by the Riemann sum: $(f(.5) + f(.5) + f(.5) + f(.5))\Delta x = .25 + 2.25 + 6.25 + 12.25 = 21$.

() **7.** $x^2 - 2x = x(x-2) = 0$ at $x = 0, 2$ and $x^2 - 2x < 0$ on $(0, 2)$, so the area of the region shown is given by $\int_0^2 -(x^2 - 2x)\,dx$

() **8.** $x(t) = \int v(t)\,dt = \int 2t - 2\,dt = t^2 - 2t + C$. $x(0) = C = 5$. The position of the object at time $t = 3$ is $x(3) = 9 - 6 + 5 = 8$.

() **9.**

a. $$\int_1^4 \frac{x+1}{\sqrt{x}}\,dx = \int_1^4 x^{1/2} + x^{-1/2}\,dx = (\frac{2}{3}x^{3/2} + 2x^{1/2})\Big|_1^4 = \frac{2}{3}(8-1) + 2(2-1) = \frac{20}{3} = 6.67.$$

b. $\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$ (let $u = 3x$, $du = 3dx$).

c. Letting $u = x^2 + 1$, $du = 2xdx$, $\int x\sqrt{x^2+1}\,dx = \int \frac{1}{2}u^{1/2}\,du = \frac{1}{3}u^{3/2} + C = \frac{1}{3}(x^2+1)^{3/2} + C$.

d. $$\int_1^e 4x^3 + \frac{1}{x}\,dx = (x^4 + \ln|x|)\Big|_1^e = e^4 - 1 + \ln e - \ln 1 = e^4.$$