SAMPLE FINAL EXAM Math 215

Study everything from the previous exams and sample exams, plus:

() 1. Evaluate the following integrals.

a. $\int_4^\infty \frac{1}{x^{5/2}}\,dx = \int_4^\infty x^{-5/2}dx = -\frac{2}{3}x^{-3/2}\Big|_4^\infty = \cancel{-\frac{2}{3}(\infty)^{-3/2}} + \frac{2}{3}4^{-3/2} = \frac{2}{3}\cdot\frac{1}{8} = \frac{1}{12}$

b. $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \int\frac{x}{2}\,dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

c. $\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$

() 2. What is the average value of the function $1/x$ on the interval [1, e]? $\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx = \frac{1}{e-1}\ln x\Big|_1^e = \frac{1}{e-1}$

() 3. Find the volume of the solid of revolution formed by rotating about the x-axis the region bounded by the curve $f(x) = x^3$, $y = 0$, $x = 0$, $x = 2$.

() 4. Solve the differential equation with the given initial condition.

a. $\frac{dy}{dt} = \frac{1}{t^2}$, $y(1) = 1$. $y = \int \frac{1}{t^2}\,dt = -\frac{1}{t} + C$ $\quad 1 = y(1) = -1 + C$ so $C = 2$

b. $\frac{dy}{dt} = 3y$, $y(0) = 4$. exp law of growth : $y = 4e^{3t}$

c. $\frac{dy}{dx} = \frac{x^2}{y}$, $y(3) = 2$. $\int y\,dy = \int x^2\,dx \Rightarrow \frac{y^2}{2} = \frac{x^3}{3} + C \Rightarrow$

$y^2 = \frac{2}{3}x^3 + 2C \qquad y = \sqrt{\frac{2}{3}x^3 + 2C}$

$2 = y(3)$

$4 = \frac{2}{3}\cdot 3^3 + 2C$

$4 = 18 + 2C$

$2C = -14$

() 5. Find the general solution for the differential equation $\frac{dy}{dx} + 2xy = x$.

$P \qquad Q$

$I = e^{\int 2x\,dx} = e^{x^2}$

$y = e^{-x^2}\int e^{x^2}x\,dx$

$= e^{-x^2}\left(\frac{1}{2}e^{x^2} + C\right)$

$= \frac{1}{2} + Ce^{-x^2}$

$\int e^{x^2}x\,dx$ let $u = x^2$, $du = 2x\,dx$

$= \int e^u \frac{1}{2}\,du$

$= \frac{1}{2}e^u + C$

$= \frac{1}{2}e^{x^2} + C$

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3) $V = \int_0^2 \pi(x^3)^2\,dx = \int_0^2 \pi x^6\,dx$

$= \pi\frac{x^7}{7}\Big|_0^2 = \pi\cdot\frac{2^7}{7} = \pi\cdot\frac{128}{7}$