4.1 Techniques for finding derivatives—continued

Notation. $y = f(x)$ **The derivative is written:** $f'(x)$, $\dfrac{dy}{dx}$ **(Leibniz notation),** $D_x y$ **or sometimes:** y', $\dfrac{d}{dx}(f(x))$ **or** $D_x(f(x))$.

Constant rule: $f(x) = k$**, a constant, for all** x**. Then** $f'(x) = 0$

Power Rule: $f(x) = x^n$ **implies** $f'(x) = nx^{n-1}$.

We showed this for n **positive integers, but in fact it is true for any real power.**

Special cases: $n = 1$ **so** $f(x) = x$ **and** $f'(x) = 1$

$n = 2$, $f(x) = x^2$ $f'(x) = 2x$

So $\dfrac{d}{dx}5x^4 = 5 \cdot 4x^3 = 20x^3$.

Suppose u **and** v **are differentiable functions.**

If $f(x) = u(x) + v(x)$**, then**

$$f'(x) = \lim_{h \to 0} \frac{u(x+h) + v(x+h) - (u(x) + v(x))}{h} =$$

$$\lim_{h \to 0} \frac{u(x+h) - u(x) + v(x+h) - v(x))}{h} = u'(x) + v'(x).$$ **Sum Rule.**

Now we can differentiate any polynomial:

Let $f(x) = 3x^{10} - 5x^2 + 12x + 4$; $f'(x) =$ **?**

Answer: $f'(x) = 30x^9 - 10x + 12$

As we said, the power rule works for any power n (negative, fraction, even irrational).

$(\sqrt{x})' = (x^{1/2})' = \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

$(\dfrac{1}{x})' = (x^{-1})' = -1 \cdot x^{-2} = -\dfrac{1}{x^2}$

$g(x) = (\sqrt{x})^3 - \dfrac{1}{x^2}$**; then** $g'(x) =$ **?**

Answer: $g(x) = x^{3/2} - x^{-2}$ **so**

$g'(x) = \frac{3}{2}x^{1/2} + 2x^{-3} = \frac{3}{2}\sqrt{x} + \dfrac{2}{x^3}$

$h(x) = \dfrac{6}{x^{1/4}}$**;** $h'(x) =$ **?**

Answer: $h(x) = 6x^{-1/4}$ **so** $h'(x) = -\frac{6}{4}x^{-5/4}$**.**

$f(x) = \dfrac{x^2 + 2x}{\sqrt{x}}$**;** $f'(x) =$ **?**

Answer: $f(x) = x^{2-\frac{1}{2}} + 2x^{1-\frac{1}{2}} = x^{3/2} + 2x^{1/2}$**, so**

$f'(x) = \frac{3}{2}x^{1/2} + x^{-1/2} = \frac{3}{2}\sqrt{x} + \dfrac{1}{\sqrt{x}}$**.**

Velocity.

$s(t) =$ **position of an object at time** t**.**

$v = \dfrac{ds}{dt}$ **is velocity; it can be positive or negative.**

$v > 0$ **means** s **is increasing (to the right; going up)**

$v < 0$ **means** s **is decreasing (to the left; going down)**

Galileo's Law: An object is dropped or thrown vertically near the surface of the earth. $s =$ **height above ground;** $s_0 =$ **initial height;** $v_0 =$ **initial velocity. Then** $s(t) = -16t^2 + v_0 t + s_0$ **(**s **measured in feet,** t **in seconds).**

A bullet is shot upward from ground level at 320ft/sec. Find $v(1), v(2)$**. When will the bullet reach its maximum height, and what is that max height? When does the bullet hit the ground?**

$s(t) = -16t^2 + 320t$**.**

$v(t) = s'(t) = -32t + 320$**;** $v(1) = 288$**,** $v(2) = 256$

The bullet reaches its max height when $v(t) = 0$**; so** $t = 10$**.** $s(10) = -1600 + 3200 = 1600 ft$**.**

The bullet hits the ground when $s(t) = 0 = -16t(t - 20)$**, so at** $t = 20$**.**

4.2 Derivatives of products and quotients.

Product rule: $\dfrac{d}{dx}(u(x)v(x)) = u(x)v'(x) + u'(x)v(x)$**.**

Examples: $\dfrac{d}{dx}[(x^3+1)(x^{-2}+x^{1/2})] =$

$(x^3+1)(-2x^{-3}+\frac{1}{2}x^{-1/2}) + (3x^2)(x^{-2}+x^{1/2})$

$\dfrac{d}{dx}[(x^3+1)^2] =$

$(x^3+1)3x^2 + 3x^2(x^3+1) = 6x^2(x^3+1)$

Reciprocal rule: $\dfrac{d}{dx}\left(\dfrac{1}{v(x)}\right) = \dfrac{-v'(x)}{v(x)^2}$**.**

Example: $\dfrac{d}{dx}\left(\dfrac{1}{x^2+1}\right) = \dfrac{-2x}{(x^2+1)^2} =$

Quotient rule: $\dfrac{d}{dx}\left(\dfrac{u(x)}{v(x)}\right) = \dfrac{v(x)u'(x)-u(x)v'(x)}{v(x)^2}$**.**

Example: $\dfrac{d}{dx}\left(\dfrac{2x+1}{3x-2}\right) =$
$\dfrac{(3x-2)2-(2x+1)3}{(3x-2)^2} = \dfrac{-7}{(3x-2)^2}$**.**

$$\frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{(x^2+1)\cdot 1 - x(2x)}{(x^2+1)^2}$$

$$\frac{d}{dx}\frac{x+1}{\sqrt{x}+1} = \frac{(\sqrt{x}+1)\cdot 1 - (x+1)(\frac{1}{2}x^{-1/2})}{(\sqrt{x}+1)^2}$$