

14. $f(x, y) = 5x^3 + 2y^2 - 60xy - 3$

$$f_x(x, y) = 15x^2 - 60y, \quad f_y(x, y) = 4y - 60x$$

$$15x^2 - 60y = 0 \quad (1)$$

$$4y - 60x = 0 \quad (2)$$

Divide equation (1) by 15 and equation (2) by 4.

$$x^2 - 4y = 0 \quad (3)$$

$$y - 15x = 0 \quad (4)$$

Solve equation (4) for y .

$$y = 15x$$

Substituting in equation (3), we have

$$x^2 - 4(15x) = 0$$

$$x^2 - 60x = 0$$

$$x = 0 \quad \text{or} \quad x = 60$$

$$y = 0 \quad \text{or} \quad y = 900.$$

$$f_{xx}(x, y) = 30x, \quad f_{yy}(x, y) = 4, \quad f_{xy}(x, y) = -60$$

At $(0, 0)$,

$$D = 0 \cdot 4 - (-60)^2 = -3,600 < 0.$$

Saddle point at $(0, 0)$

At $(60, 900)$,

$$\begin{aligned} D &= 1,800 \cdot 4 - (-60)^2 \\ &= 3,600 > 0 \quad \text{and} \quad f_{xx}(x, y) > 0. \end{aligned}$$

Relative minimum at $(60, 900)$