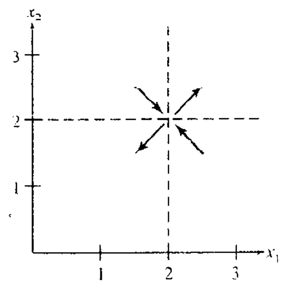
11.5 Nonlinear Systems of Different Equations

2. $$\frac{dx_1}{dt} = 3x_1^2x_2 - 6x_1^2$$
$$\frac{dx_2}{dt} = x_1x_2^2 - 2x_2^2$$

Find the equilibrium point.

$$\frac{dx_1}{dt} = 0 = 3x_1^2x_2 - 6x_1^2 \quad , \quad \frac{dx_2}{dt} = 0 = x_1x_2^2 - 2x_2^2$$
$$3x_1^2x_2 = 6x_1^2 \quad , \quad x_1x_2^2 = 2x_2^2$$
$$x_2 = 2 \quad , \quad x_1 = 2$$

region	1	2	3	4
dx_1/dt	+	+	−	−
dx_2/dt	+	−	−	+



8. (a) $$\frac{dx_1}{dt} = x_1^2x_2 - x_1^3$$
$$\frac{dx_2}{dt} = 6x_2^2 + x_2^3 - 3x_1x_2^2$$

Find the equilibrium point.

$$\frac{dx_1}{dt} = 0 = x_1^2x_2 - x_1^3$$

$x_2 = x_1$ eq.(1),

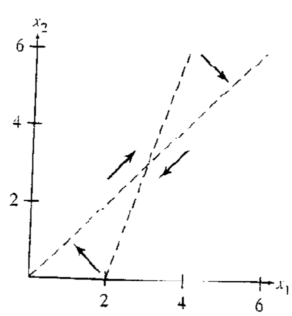
$$\frac{dx_2}{dt} = 0 = 6x_2^2 + x_2^3 - 3x_1x_2^2$$

$$0 = 6 + x_2 - 3x_1$$
$x_2 = 3x_1 - 6$ eq.(2),

Solving equations (1) and (2) simultaneously gives

$$x_1 = 3, x_2 = 3.$$

(b) Plot equations (1) and (2) and test each region.



10. (a) $$\frac{dx_1}{dt} = -3x_1 + 4x_1x_2$$
$$\frac{dx_2}{dt} = -3x_2 + x_1x_2$$

$$\frac{dx_2}{dx_1} = \frac{\frac{dx_2}{dt}}{\frac{dx_1}{dt}} = \frac{-3x_2 + x_1x_2}{-3x_1 + 4x_1x_2} = \frac{x_2(-3 + x_1)}{x_1(-3 + 4x_2)}$$

Separating variables yields

$$\frac{-3 + 4x_2}{x_2}dx_2 = \frac{-3 + x_1}{x_1}dx_1$$

or,

$$\int \left(\frac{-3}{x_2} + 4\right) dx_2 = \int \left(\frac{-3}{x_1} + 1\right) dx_1$$
$$-3\ln x_2 + 4x_2 = -3\ln x_1 + x_1 + C$$

Use the initial condition $x_2 = 1$ when $x_1 = 3$ to find C.

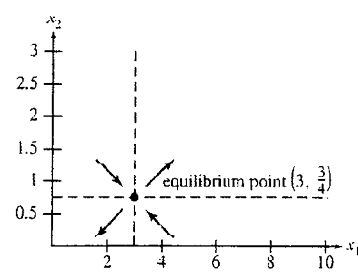
$$-3\ln 1 + 4(1) = -3\ln 3 + 3 + C$$
$$0 + 4 = -\ln 27 + 3 + C$$
$$C = 1 + \ln 27$$

The desired equation is

$$3\ln x_1 - x_1 - 3\ln x_2 + 4x_2 = 1 + \ln 27.$$

(b) $$\frac{dx_1}{dt} = 0 = -3x_1 + 4x_1x_2$$
$$x_2 = \frac{3}{4}$$
$$\frac{dx_2}{dt} = 0 = -3x_2 + x_1x_2$$
$$x_1 = 3$$

(c), (d)



The figure shows that if both populations are greater than their equilibrium values they will increase without bound and if they are less than their equilibrium values they will decrease toward zero.