A huge collection of low-value Magic cards appraised at $100 is being divided by 5 kids ($P_1, P_2, P_3, P_4, P_5$) using the last-diminisher method. The players play in a fixed order, with $P_1$ first, $P_2$ second, and so on. In round 1, $P_1$ makes the first selection and makes a claim on a pile of cards. For each of the remaining players, the value of the current pile of cards at the time it is their turn is given in the following table:

<table>
<thead>
<tr>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8$</td>
<td>$15$</td>
<td>$22$</td>
<td>$18$</td>
</tr>
</tbody>
</table>

1. Which player gets his or her share at the end of round 1?
   - A. $P_1$
   - B. $P_2$
   - C. $P_3$
   - D. $P_4$
   - E. $P_5$

   Note that $\text{Appraised value/\# of players} = \frac{100}{5} = 20$, so that's everyone's target for fairness.

   Player 1's original cut would have valued the pile at $20$.
   Player 2 thought the pile was worth only $15$, so passes.
   Player 3 thinks the pile is worth $22$, so trims it down to $20$.
   Players 4 and 5 already had seen the pile as worth $<20$, now after trimming see it as worth even less, so they both pass.
   Since player 3 was the last player to trim the pile, s/he gets it.

2. What is the value of the share to the player receiving it?
   - A. $8$
   - B. $15$
   - C. $22$
   - D. $18$
   - E. $19$

   As mentioned, Player 3 trimmed the pile to what s/he thought was a value of $20$.

3. 7 players are dividing a cake using the Fink protocol (also known as Method of Lone Chooser). After the first six rounds the first 6 players each have what they believe to be 1/6 of the cake. At this next (seventh) round, a bunch of cuts are going to be made. How many in total?
   - A. 6
   - B. 36
   - C. 42
   - D. 49

   Each of the 6 players cuts their slice into 7 pieces, so $42$ pieces altogether.

Three players, $A$, $B$, and $C$, want to divide the objects in the figure below using the method of markers. Their markers have already been placed.

4. Who gets the first group of objects?  
   - A. $A$
   - B. $B$
   - C. $C$

5. How many pieces does $B$ get (before leftovers are divided up)?  
   - A. 18
   - B. 8
   - C. 11
   - D. 12

6. How many leftover pieces are there?  
   - A. 0
   - B. 1
   - C. 2
   - D. 3
   - E. 4

One way to start is with a table:

<table>
<thead>
<tr>
<th>A</th>
<th>1st group</th>
<th>B</th>
<th>2nd group</th>
<th>C</th>
<th>3rd group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td></td>
<td>10-18</td>
<td></td>
<td>19-30</td>
<td></td>
</tr>
</tbody>
</table>
The first "1st group" is A's (ends soonest), so A gets 1-9.

The first "2nd group" is B's (ends at 15 which is less than C's 20), so we don't consider A any more since he has his share.

So B gets 11-18 (8 pieces)

Finally, C gets his 3rd group, which is 21-30.

Left over are 10, 19, and 20 (3 pieces)

As part of an inheritance, four children, Abby, Ben, Carla, and Dan, are dividing four vehicles using Sealed Bids. Their bids (in thousands of dollars) for each item is shown below. Use the Sealed Bid method to allocate the goods; answer the given questions about the final allocation.

<table>
<thead>
<tr>
<th></th>
<th>Abby</th>
<th>Ben</th>
<th>Carla</th>
<th>Dan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorcycle</td>
<td>6</td>
<td>7</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Car</td>
<td>8</td>
<td>13</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Tractor</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Boat</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

7. Who inherits the car? A. Abby  B. Ben  C. Carla  D. Dan

8. What is the total cash value of Ben's inheritance (in thousands)?
   A. 8  B. 9  C. 10  D. 11  E. None of the above

9. Who inherits the most (total cash value)? A. Abby  B. Ben  C. Carla  D. Dan

The high bid for the motorcycle was Carla, so she inherits it.

Let's indicate this on the table:

<table>
<thead>
<tr>
<th></th>
<th>Abby</th>
<th>Ben</th>
<th>Carla</th>
<th>Dan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motorcycle</td>
<td>6</td>
<td>7</td>
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<td>8</td>
</tr>
<tr>
<td>Car</td>
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<td>3</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Boat</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Fair Market Share = \(\frac{\text{sum of bids}}{\text{number of items}}\)

\[
\begin{align*}
\text{Motorcycle} & = \frac{6 + 7 + 10 + 8}{4} = \frac{21}{4} = 5.25 \text{ Thousand dollars} \\
\text{Car} & = \frac{8 + 13 + 10 + 12}{4} = \frac{43}{4} = 10.75 \text{ Thousand dollars} \\
\text{Tractor} & = \frac{3 + 2 + 5 + 4}{4} = \frac{14}{4} = 3.5 \text{ Thousand dollars} \\
\text{Boat} & = \frac{7 + 6 + 3 + 8}{4} = \frac{24}{4} = 6 \text{ Thousand dollars}
\end{align*}
\]

Estate now has 6 + 6 + 8 + 10 = 20 Thousand dollars, divided 4 ways = 5 Thousand

So Abby inherits no goods + 6 (the 6 she owes the estate) + 2 (estate 14442) = 8

Ben inherits 13 - 6 (amount owed to estate) + 2 = 9

(By the way, if you have trouble understanding the part of these calculations, think of it this way: Ben got a car worth 13; but the fair market share of the estate based on his own valuations is only 7, that's why he
owes the estate 13-7=6, so he's left with (13-(13-7))=7 + whatever his share of the leftovers is (2).

Carl inherits 15-8+2 = 9

Dan inherits 8-0+2 = 10, The most

10. A cake is to be divided among Roy, Sam, and Trish by the Selfridge-Conway variant of Lone Divider.

The three players value the pieces after the first cut as follows:

<table>
<thead>
<tr>
<th></th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roy</td>
<td>10%</td>
<td>55%</td>
<td>35%</td>
</tr>
<tr>
<td>Sam</td>
<td>33.33%</td>
<td>33.33%</td>
<td>33.33%</td>
</tr>
<tr>
<td>Trish</td>
<td>40%</td>
<td>30%</td>
<td>30%</td>
</tr>
</tbody>
</table>

Who divides the trimmings? (This is after the trimmed pieces are already allocated to the diners.)

A. Roy  B. Sam  C. Trish

(Both are correct!)

1) Sam is the initial divider (not of the trimmings) since the initial divider cuts the cake into equal pieces by his reasoning.

2) The 1st chooser, Roy, trims P2 down to 35%, pus the rest away.

3) Trish now chooses P1, obviously.

4) Roy's now up, he takes trimmed piece P2

5) Sam gets P3

6) Now Trish is the chooser who got the untrimmed piece, so Trish divides the trimmings.

(Note: After Sam's initial division we could have decided that Trish was "Chooser A" in the algorithm and Roy was "Chooser B". In that case, Trish would have trimmed P1 down to 30%, Roy would have chosen P2, Trish P1, and now Roy does the dividing of the trimmings, in other words, this exercise has 2 correct answers!)