Math 471 Homework Assignment 1

1. Let \((S, \mathcal{A}, P)\) be a probability space, where \(\mathcal{A}\) is the \(\sigma\)-field of all subsets of \(S\) and \(P\) is a probability measure that assigns probability \(p > 0\) to each one-point subset of \(S\) (i.e. every outcome is equally likely).

(a) Show that \(S\) must be a finite set.
(b) Let \(n\) be the number of elements in \(S\). Show that \(p = 1/n\).

2. Let \((S, \mathcal{A}, P)\) be a probability space. Let \(A_n, n = 1, 2, 3, \ldots\), be a countable collection of events such that

\[ A_1 \subset A_2 \subset A_3 \subset \cdots \subset A_n \subset \cdots. \]

Prove that

\[
P \left( \bigcup_{j=1}^{\infty} A_j \right) = \lim_{n \to \infty} P \left( \bigcup_{j=1}^{n} A_j \right) . \tag{1}
\]

Hint: consider the sets \(B_1, B_2, B_3, \ldots\), where \(B_1 = A_1\) and \(B_k = A_k - A_{k-1}\) for \(k = 2, 3, 4, \ldots\).

3. Let \(A_n, n = 1, 2, 3, \ldots\), be a countable collection of events such that

\[ A_1 \supset A_2 \supset A_3 \supset \cdots \supset A_n \supset \cdots. \]

Use equation (1) to prove that

\[
P \left( \bigcap_{j=1}^{\infty} A_j \right) = \lim_{n \to \infty} P \left( \bigcap_{j=1}^{n} A_j \right) . \]