

Problems of the Month for UH Mānoa Undergraduates

Problems for March 2008

Problem A. Let $x_1 = 1$, and for an integer $n \geq 1$, let

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{2}{x_n} \right),$$

so that $x_2 = \frac{1}{2} \left(x_1 + \frac{2}{x_1} \right) = 3/2$, $x_3 = \frac{1}{2} \left(x_2 + \frac{2}{x_2} \right) = \frac{1}{2} \left(3/2 + \frac{2}{3/2} \right) = \frac{17}{12}$, etc.

(a) Prove that

$$\lim_{n \rightarrow \infty} x_n = \sqrt{2},$$

i.e., for any $\varepsilon > 0$, there is some integer $N > 0$, such that $|x_n - \sqrt{2}| < \varepsilon$ for $n > N$.

(b) Assume now that $x_1 = -1$ and we still have $x_{n+1} = \frac{1}{2} \left(x_n + \frac{2}{x_n} \right)$. Now find $\lim_{n \rightarrow \infty} x_n$ and prove that your answer is correct.

Problem B. Suppose that every point in the xy -plane is colored red, yellow or blue. Show that there are 2 points of the same color which are exactly 1 unit apart.

Rules

The following rules may be changed or clarified from month to month.

1. Any “regular” undergraduate currently enrolled at UH Manoa is eligible to compete.
2. Write a complete solution with all details to either problem or both.
3. Submit your solution(s) *electronically* before the end of the above month to

bleecker@math.hawaii.edu

For the subject line of your email use “problem of the month” and *send your email via your UH email address*. Either write your solution within the

body of your email or within attachment(s) in the form of readable pdf files or images of your work in jpg format (e.g., scanned or digitally photographed).

4. Solutions will be judged by a committee of professors according to a combination of criteria: accuracy, attention to details, chronological order of submission, and neatness, but not necessarily in that order.

5. Before the end of the 10-th day of the month that follows, the winner(s) will be announced on the Math Department web site. Moreover, if there is at least one good answer to a problem the winner(s) for that problem will collectively receive a total of least \$20 to be distributed among them depending on the criteria in 4 above, as soon as the checks can be extracted from the Hanf Fund at the UH Foundation, a process that may take several weeks.