MULTIPLE SOLUTIONS FOR A NONLINEAR DIRICHLET PROBLEM VIA MORSE INDEX

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ABSTRACT

In this talk we study the existence of multiple solutions for the nonlinear Dirichlet problem

\[
\begin{align*}
\Delta u + f(u) &= 0 \quad \text{in } \Omega, \\
u &= 0 \quad \text{on } \partial\Omega,
\end{align*}
\]

where $\Omega$ is a smooth bounded region in $\mathbb{R}^N$, $\Delta$ is the Laplacian operator, and the nonlinearity $f : \mathbb{R} \to \mathbb{R}$ is asymptotically linear, i.e.,

\[
f'(\infty) := \lim_{|t| \to \infty} f'(t) \in \mathbb{R}.
\]

We prove that problem (1) has at least five solutions when the range of the derivative of the nonlinearity includes at least the first four eigenvalues. Extensive use is made of Lyapunov-Schmidt reduction arguments, the invariance of the Morse index under Lyapunov-Schmidt reduction method, the mountain pass lemma, and characterizations of the local degree of critical points. Our result extends the main theorem in [1].


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