Crossbars are Fun!

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Outline

• Crossbars
• Shannon’s Lower Bound on the Number of 2x2 Switches
  – Lower bounds can be useful
• Construction of an Efficient Crossbar: Benes Network
• Summary
Simple Switch Fabrics
Crossbar

Crosspoint
(2x2 switch)

n inputs

n outputs

nxn crosspoints
Simple Switch Fabric
Crossbar

- **Nonblocking**: Given a request to connect the inputs to the outputs in an arbitrary way (but one input per output and vice versa)

  the switch can realize the connections.

  The crossbars are nonblocking
Another Simple Switch Fabric
Bus Crossbar

n inputs

m outputs

n:1 Multiplexer

Bus
Candidate Crossbar
Omega Network

At Stage k:
- \( b_k = 0 \): route up
- \( b_k = 1 \): route down

Routing
Dest addr
\( b_2b_1b_0 \)
Omega Network

Restrictions
# Inputs/outputs must be a power of 2

Forms a tree

How many switches?

Is it nonblocking?
Shannon’s Lower Bound

A network of 2x2 switches

S = Number of switches
How many do we need?

# States <= 2^S

S >= log2 (# States)
Shannon’s Lower Bound

This is a state

We can represent this state by a permutation (1,2,0,3) of the inputs

\# States \geq \# permutations of inputs

= n!
Shannon’s Lower Bound

\[ n! \approx \sqrt{2\pi n^{n+1/2} e^{-n}} \]

Stirling’s Approximation
(my crude version)

\[ S \geq \log_2 (n!) = (n + 1/2) \log_2 n - n \log_2 e + \text{smaller stuff} = (1 + o(1)) \times n \log_2 n \]
Omega Network
Is it nonblocking?

How many switches?
Another Switch Network
Benes Network

• A three stage Clos switching network
  – Has n inputs and n outputs
    • n is a power of 2
  – First and third stage is a column of 2x2 switches
  – Second stage is a column of two nonblocking switches
Clos Network: 3 Stages

Upper Nonblocking Switch

Lower Nonblocking Switch

n inputs

Etc

n outputs

Etc
Is it Nonblocking?

• Given an arbitrary request of connections from the inputs to the outputs, can the request always be realized?
  – Can the connections be routed from inputs to outputs?
  – Yes, using a “looping algorithm”
  – We’ll use a bipartite graph representation of the set of connection requests
Benes Network

Arbitrary Connection Request

n inputs

n outputs
Benes Network

Degree 2

Find a tour

Start at an arbitrary node, keep visiting new edges (which are connections) until you return to the initial node.
Benes Network

Degree 2

Find tour

Start at an arbitrary node, keep visiting new edges until you return to the initial node
Benes Network

Degree 2

Find an tour
Benes Network

Degree 2

Tour
Benes Network

Degree 2

Extract Tour and find another tour
Benes Network

Degree 2

Extract Tour and find another tour
Routing Connections with Euler Tours

Alternate labels of edges between Upper and Lower
Routing Connections
Routing Algorithm for Connections

• Find tours until all edges are covered

• Label edges of tours “upper” and “lower”
  – Note each node has an upper edge and a lower edge

• Route connections/edges according to the labels
  – Routes are feasible since each 2x2 switch is connected to the upper switch and lower switch
Apply 3 Stage Architecture
To Middle Switches
Keep Applying 3 Stage Architecture Until Nothing But 2x2 Switches

Benes: \# 2x2 Switches = \(\frac{n}{2} \times (2\log_2 n - 1)\) \{stages\}
= approx. \(n \log_2 n\)

How good is this? Close to Shannon’s lower bound
Summary

• Crossbars and Nonblocking Networks
• Shannon’s Lower Bound
• Benes Network Nearly Achieving the Lower Bound