Spring 2000

Math 253 – ACCELERATED CALCULUS III (4)

Vector calculus; maxima and minima in several variables; multiple integrals; line integrals, surface integrals and their applications.

Pre: Math 252

Text: *Vector Calculus* by Susan Jane Colley.

Chapter 2: Differentiation in Several Variables. (3 weeks)
Functions of several variables and graphing surfaces. The topology of $\Re^n$, limits and continuity are discussed in Section 2.2. The next three sections cover derivatives and differentiability, partial derivatives and higher order partials, and the chain rule, all for functions $f : \Re^n \longrightarrow \Re^m$ but with emphasis on the scalar–valued case $(m = 1)$. Section 2.6 introduces the gradient and directional derivatives for scalar–valued functions, and could be done immediately after Section 2.3.

Chapter 3: Vector–Valued Functions. (3 weeks)
The first half of this chapter covers parameterized curves in $\Re^n$. The topics include the fundamental notions of differential geometry: velocity and acceleration, tangent and normal vectors, arclength and curvature. The second half covers vector fields $F : \Re^n \longrightarrow \Re^n$, with emphasis on the case $n = 3$. Topics include the divergence, gradient and curl of a vector field. The Del operator is introduced.

Chapter 4: Maxima and Minima in Several Variables. (3 weeks)
Review Taylor's Theorem and differentials in one variable. (Again, do not assume that the students remember these.) The text then covers Differentials and Taylor's Theorem (with remainder) for scalar–valued functions of several variables. These are applied to the problem of finding and classifying extrema, including the second derivative (Hessian) test and the Extreme Value Theorem for compact subsets of $\Re^n$. Lagrange multipliers are used for constrained optimization problems. The chapter concludes with applications of the methods to least squares and physical equilibrium problems.

Chapter 5: Multiple Integration. (2 weeks)
Double and Triple integrals in rectangular coordinates. Changing the order of integration: Fubini's theorem and how to do it in practice. Change of variables is covered both abstractly (using the Jacobian) and concretely: double integration in polar coordinates, triple integration in cylindrical and spherical coordinates. Applications include the average value of a function, center of mass, and moments of inertia.

Chapter 6: Line Integrals. (2 weeks)
Section 6.1 covers both scalar line integrals (integration with respect to arclength) and vector line integrals. The latter are applied to work problems. The independence of both under (orientation–preserving) reparameterization is discussed and proved. Section 6.2 covers Green's Theorem and the divergence theorem. Section 6.3 deals with independence of path for line integrals and conservative vector fields on simply connected regions in $\Re^2$ or $\Re^3$.

Chapter 7: Surface Integrals and Vector Analysis. (2 weeks)
Scalar and vector surface integrals for smooth parameterized surfaces in $\Re^3$. The independence of both under (orientation–preserving) reparameterization. After defining orientable surfaces, the text covers Stokes's and Gauss's Theorems. These topics are fairly abstract, so the treatment should include lots of straightforward examples. There is a short general introduction to differential forms in Section 7.5. If time permits, Green's formulas and Maxwell's equations in Section 7.4 provide a concrete application of these results.