Math 242 Power Series Lab

Given a function, \( f(x) \), with derivatives of all orders, the Taylor Series generated by \( f \) at \( x = a \) is:

\[
\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!} (x - a)^k = f(a) + f'(a)(x - a) + \frac{f''(a)}{2!} (x - a)^2 + \cdots + \frac{f^{(n)}(a)}{n!} (x - a)^n + \cdots
\]

The Taylor Polynomial of order \( n \) is the Taylor Series truncated at the \( n \)-th term.

The Remainder Estimation Theorem:

\[
|R_n(x)| \leq M \frac{|x - a|^{n+1}}{(n + 1)!}
\]

where \( M \geq |f^{(n+1)}(t)| \) for all \( t \) between \( x \) and \( a \), inclusive.

1a) Find the 4th order Taylor polynomial generated by \( f(x) = \cos(x) \) at \( a = 0 \). Use your polynomial to approximate \( \cos(0.2) \).

b) Use the Remainder Estimation Theorem to find an error bound for your approximation of \( \cos(0.2) \). (Round to three significant figures)

2a) Find the 3rd order Taylor polynomial generated by \( f(x) = \sqrt{x} \) at \( a = 9 \). Use your polynomial to approximate \( \sqrt{8.8} \).

b) Use the Remainder Estimation Theorem to find an error bound for your approximation of \( \sqrt{8.8} \).

3) Find a Taylor polynomial for \( f(x) = \ln(x+1) \) that will approximate \( \ln(1.3) \) to within 0.001 error?