Math 252: Numerical Integration Techniques

**LRAM**
\[ \sum_{i=0}^{n-1} f(x_i)\Delta x = \Delta x[f(x_0) + f(x_1) + \cdots + f(x_{n-2}) + f(x_{n-1})] \]

**RRAM**
\[ \sum_{i=1}^{n} f(x_i)\Delta x = \Delta x[f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + f(x_n)] \]

**MRAM**
\[ \sum_{i=0}^{n-1} \left( f(x_i) + f(x_{i+1}) \right) \frac{\Delta x}{2} = \Delta x \left[ \frac{f(x_0 + x_1)}{2} + \frac{f(x_1 + x_2)}{2} + \cdots + \frac{f(x_{n-1} + x_n)}{2} \right] \]

**Trapezoidal**
\[ \sum_{i=0}^{n-1} \left( \frac{f(x_i) + f(x_{i+1})}{2} \right) \Delta x = \frac{\Delta x}{2} \left[ f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \]

**Simpson’s**
\[ \sum_{i=1, \text{odd}}^{n-1} \frac{1}{3} \left( f(x_{i-1}) + 4f(x_i) + f(x_{i+1}) \right) \Delta x = \frac{\Delta x}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right] \]

**Using Maxima**
(tutorial: http://www.math.hawaii.edu/~dave/wxMaxima/num_int_graphics_demo2.pdf)

\begin{align*}
\text{a:1} \: b:2 \: f(x):=1/x & \quad \text{Defines starting point a=1, ending point b=2, and function f=1/x} \\
x(i,n):= a + i*((b-a)/n); & \quad \text{Defines function x which will output } x_i \\
\text{sum(f(x(i,n)),i,1,n)} & \quad \text{i is the index, n is the number of partitions} \\
\text{sum(f(x(i,n)),i,1,n) * ((b-a)/n);} & \quad \text{Sums function f, over index i, from } i=1 \text{ to } i=n
\end{align*}

**Example: RRAM:**
\[ \text{right(n):= sum(f(x(i,n)),i,1,n) * ((b-a)/n);} \]
1) Consider the integral \( \int_{0}^{6} (-x^2 + 36) \, dx \)

Input the function into Maxima with the following line of code: \( f(x) := -x^2 + 36 \)

a) Use LRAM with \( n = 6 \) to approximate the integral.
b) Use RRAM with \( n = 6 \) to approximate the integral.
c) Use Trapezoidal rule with \( n = 6 \) to approximate the integral.
d) Evaluate the integral (you can use maxima’s integrate function). Which method was most accurate?

2) Consider the integral \( \int_{0}^{5} (x^2 + 6)^{1/3} \, dx \)

The Maxima input for the function is: \( f(x) := (x^2 + 6)^{1/3} \)

a) Graph the function \( f(x) = (x^2 + 6)^{1/3} \) on \( 0 \leq x \leq 5 \). Sketch the rectangles for MRAM with \( n = 5 \).
b) Use MRAM with \( n = 5 \) to approximate the integral.
c) Graph the function \( f(x) = (x^2 + 6)^{1/3} \) on \( 0 \leq x \leq 5 \). Sketch the trapezoids for trapezoidal rule with \( n = 5 \).
d) Use Trapezoidal rule with \( n = 5 \) to approximate the integral.

3) Consider the integral \( \int_{1}^{7} \ln x \, dx \).

Use Simpson’s rule with \( n = 6 \) to approximate the integral. (note: “\( \ln x \)” is represented by \( \log(x) \) in Maxima)

4) Consider the integral \( \int_{0}^{6} x^2 + 6x - 16 \, dx \)

The following lines of code will define Simpson’s rule in Maxima for this integral:

\[
a := 0 \quad b := 6 \quad f(x) := x^2 + 6x - 16 \\
x(i, n) := a + i * ((b-a)/n); \\
simp(n) := \text{sum}(f(x(2*i-2, n)) + 4*f(x(2*i-1, n)) + f(x(2*i, n)), i, 1, n/2) * ((b-a)/n) * (1/3);
\]

a) Use Simpson’s rule with \( n = 2 \) to approximate the integral
b) Use Simpson’s rule with \( n = 6 \) to approximate the integral
c) Use Simpson’s rule with \( n = 12 \) to approximate the integral
d) Evaluate the integral. Explain your results.

Next week: Error estimates with numerical integration