For your reference:

**n-th Term Test**: If \( \lim_{n \to \infty} a_n \neq 0 \), then \( \sum a_n \) diverges.

**Geometric Series**: Converges if \( |r| < 1 \):

\[
\sum a_n = \frac{a_1}{1 - r}
\]

**Integral Test**: If \( f(n) = a_n \) for a continuous, positive, decreasing function of \( x \) for all \( x \geq N \), then \( \int_N^\infty f(x) \) and \( \sum_{n=N}^\infty a_n \) both converge or both diverge.

**Comparison Test**: (Analogous to the Direct Comparison Test for Integrals)
If \( \sum a_n \) is a series with non-negative terms, then
(a) \( \sum a_n \) converges if there is a convergent series \( \sum c_n \) with \( a_n \leq c_n \) for all \( n > N \).
(b) \( \sum a_n \) diverges if there is a divergent series \( \sum d_n \) with \( 0 \leq d_n \leq a_n \) for all \( n > N \).

**Limit Comparison Test**: (Analogous to the Limit Comparison Test for Integrals)
Suppose \( a_n > 0 \) and \( b_n > 0 \) for all \( n > N \).
(a) If \( \lim_{n \to \infty} \frac{a_n}{b_n} = c > 0 \), then \( \sum a_n \) and \( \sum b_n \) both converge or both diverge.
(b) If \( \lim_{n \to \infty} \frac{a_n}{b_n} = 0 \) and \( \sum b_n \) converges, then \( \sum a_n \) converges.
(c) If \( \lim_{n \to \infty} \frac{a_n}{b_n} = \infty \) and \( \sum b_n \) diverges, then \( \sum a_n \) diverges.

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**Coding in Maxima**:

Example: Suppose we want to find the first 10 terms of the sequence of partial sums given by:

\[
\sum_{n=1}^k \frac{1}{n}
\]

i) We define a function for the partial sum

\( g(y) := \text{sum}(1/x, x, 1, y) \);

ii) We make a list of the first 10 terms:

\( \text{makelist}(g(y), y, 1, 10) \);
iii) If we want to plot the first 10 terms, we first make a list of the discrete points:
pts : makelist([y,g(y)],y,1,10);

iv) Plot the points as a discrete plot:
wxplot2d([discrete,pts],[style,points]);

1a) By the integral test, \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges (despite the fact that \( \frac{1}{n} \) converges to 0). Tweak the code in the above example to plot the first 100 partial sums to verify this. Sketch the plot (you don’t have to draw all 100 points).

b) Tweak the code to plot the first 100 partial sums of:
\[
\sum_{n=1}^{k} \frac{1}{n^2}
\]
Sketch your plot.

(By the integral test, \( \sum_{n=1}^{\infty} \frac{1}{n^2} \) converges – compare your graphs in parts (a) and (b); but keep in mind that a graph is NOT proof of convergence or divergence).

2) We will demonstrate the above point. Run the following lines of code which will graph the 200th-220th partial sums of
\[
\sum_{n=2}^{k} \frac{1}{n \ln(n)}
\]
g(y):=sum(1/(x*log(x)),x,2,y);
pts : makelist([y,g(y)],y,200,220);
wxplot2d([discrete,pts],[style,points],[y,2,3]);

a) Sketch the graph.

b) Despite the appearance of the graph, the series diverges. Use the integral test (by hand) to prove that \( \sum_{n=2}^{\infty} \frac{1}{n \ln(n)} \) diverges.

3) Determine (by hand) whether the following series diverge or converge. Give careful reasons for your answers.

a) \( \sum_{n=1}^{\infty} \frac{n^n}{n!} \)

b) \( \sum_{n=1}^{\infty} \frac{9^n}{8^n - 18} \)
c) \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 1} \)

d) \( \sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2 - 1}} \)

e) \( \sum_{n=1}^{\infty} \frac{1 + 4 \sin(n)}{n \sqrt{n}} \)