Problem 1 Consider an arbitrary $m \times n$ matrix $A$ and arbitrary $n \times p$ matrices $B$ and $C$. Show that $A(B+C) = AB + AC$. 
Problem 2  Find the null space of the matrix
\[
\begin{bmatrix}
0 & 2 & -1 & 1 & 0 \\
2 & 2 & 0 & 0 & -1 \\
-2 & 0 & -1 & 1 & 1
\end{bmatrix}.
\]
Problem 3 Find the null space of the matrix

\[
\begin{bmatrix}
2 & 2 & 3 & 4 \\
0 & 1 & \frac{1}{2} & 1 \\
1 & 0 & 1 & 1 \\
-1 & 0 & 0 & -1
\end{bmatrix}
\]
Problem 4 Let $A$ be an arbitrary $m \times n$ matrix. Find a matrix $C$ such that $CA = B$ for each of the following matrices $B$.

a. $B$ is the matrix that results from multiplying row $i$ of $A$ by a nonzero number $c$.

b. $B$ is the matrix that results from swapping rows $i$ and $j$ of $A$.

c. $B$ is the matrix that results from adding $c$ times row $i$ of $A$ to row $j$ of $A$. 