Problem 1  Let $A$ be an $m \times n$ matrix and $B$ be an $n \times p$ matrix.

1. Give an example of $A$ and $B$ such that $\text{null}(AB) \supseteq \text{null}(B)$.

2. Give an example of $A$ and $B$ such that $\text{null}(AB) = \text{null}(B)$.

3. Prove that $\text{null}(AB) \supseteq \text{null}(B)$. 
Problem 2 Suppose $A$ and $B$ are $n \times n$ matrices, that $\det(B) \neq 0$ and there are $k$ linearly independent vectors in $\text{null}(A)$ for some $k \leq n$. Show that there are $k$ linearly independent vectors in $\text{null}(AB)$. 
Problem 3  Suppose that $S = \{v_1, \ldots, v_n\}$ is a linearly independent set of vectors in a vector space $V$. Show that $T = \{a_1v_1, \ldots, a_nv_n\}$ is linearly independent for any nonzero numbers $a_1, \ldots, a_n$. 