Problem 1  Give examples of linear transformations that satisfy each of the following descriptions.

1. \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) such that \( T \) is one-to-one.

2. \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) such that \( T \) is not onto.

3. \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^4 \) such that \( T \) is not one-to-one.
Problem 2 Suppose that $T : U \rightarrow V$ and $S : V \rightarrow W$ are linear transformations, where $U$, $V$ and $W$ are vector spaces. Prove that $S \circ T$ is a linear transformation.
Problem 3 Suppose that $T : U \rightarrow V$ is a linear transformation, where $U$ and $V$, and there is a non-zero vector, $u$, in the null space of $T$. Prove that $T$ is not one-to-one.
Problem 4 Find a matrix, $B$, such that $T(x) = Bx$, where $T$ is the unique linear transformation such that

\[
T\left( \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ \frac{1}{2} \\ -1 \end{bmatrix}, \quad T\left( \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}, \quad T\left( \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]