Problem 1 Find the determinant of

\[
A = \begin{bmatrix}
2 & 2 & 0 & 2 \\
1 & -1 & 0 & 0 \\
2 & 0 & -1 & 1 \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

Also, find the determinant of \(A^6 = AAAAAA\).

Solution The determinant of \(A\) is \(-2\). Also, the determinant of \(A^6\) is \((-2)^6 = 64\).

\[
\square
\]

Problem 2 Prove that if \(A\) is not invertible, then \(A^n\) is not invertible for any \(n \in \mathbb{N}\). (Hint: Think about determinants.)

Solution If \(A\) is not invertible, then \(\det(A) = 0\). Thus,

\[
\det(A^n) = (\det(A))^n = 0^n = 0.
\]

Therefore, \(A^n\) is not invertible.

\[
\square
\]