

Elementary Probability and Statistics
Midterm II Solutions
May 20, 2011

↑ Student name and ID number ↑

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Disclaimer: It is essential to write legibly and *show your work*. If your work is absent or illegible, and at the same time your answer is not perfectly correct, then no partial credit can be awarded. Completely correct answers which are given without justification may receive little or no credit.

During this exam, you are not permitted to use notes, or books, nor to collaborate with others. You are allowed to use a calculator.

Score: _____

Problem 1. Suppose students enter the University of Phoenix psychology PhD program at a rate of 10 per year on average. Among these students, 60% are women.

- (a) (7 points) What is the probability that exactly 4 out of the next 10 students entering the program are women?

Solution: $\binom{10}{4}(.6)^4(1 - .6)^{10-4} = 0.111476736 = 11.1\%$ (binomial distribution).

- (b) (6 points) What is the probability that exactly 9 students enter the program in the next two years?

Poisson distribution: $\lambda = 2 \cdot 10 = 20$, $k = 9$, probability is

$$e^{-\lambda} \frac{\lambda^k}{k!} = e^{-20} \frac{20^9}{9!} = 0.0029 = 0.29\%.$$

- (c) (6 points) What is the probability that it will be between 1 and 2 months before the next student enters the program? (Assume that students can enter the program at any time, i.e., not necessarily at the beginning of a term.)

X , the time until the next student enters the program, should have an exponential distribution with parameter $\lambda = 10$. Then

$$\mathbb{P}(1/12 \leq X \leq 2/12) = \int_{1/12}^{2/12} 10e^{-10x} dx = \frac{e^{5/6} - 1}{e^{5/3}} = .245723 = 24.6\%.$$

Score: _____

Problem 2. [18 points] For each of the random variables below, indicate whether the distribution would be best described as binomial, geometric, Poisson, exponential, uniform, or normal. You may use an answer more than once.

- a) The number of papers, out of the next 10 published by Professor Li, that have an even number(2,4,6,8, etc.) of occurrences of the word “and”. **Binomial (with $n = 10$ and $p = 1/2$).**
- b) The number of emails that a person receives in a day. **Poisson.**
- c) The proportion who vote for Obama among 100 randomly selected voters in the U.S. **Normal (or binomial, divided by 100).**
- d) The number of times you have to toss a coin before getting a tail. **Geometric.**
- e) The amount of time before the next paper is published by Professor Li. **Exponential.**
- f) The sum of the weights of 10,000 randomly chosen babies. **Normal.**

Problem 3. *Bounce rate* is an Internet marketing term used in web traffic analysis. It represents the percentage of visitors who enter the site and “bounce” (leave the site) rather than continue viewing other pages within the same site. Suppose that for the web site *math.hawaii.edu*, the overall bounce rate is 55.23% on average.

On some recent weekend days the average bounce rate was observed to be 58.94%. The total number of visitors on the observed days was 1,792. Do these results provide strong evidence that the bounce rate is higher on weekends?

- a) [6 pts] State appropriate null and alternative hypotheses.

Let b_W denote the bounce rate on weekends. Then the null hypothesis H_0 states that $b_W = 55.23\%$. The alternative hypothesis H_A states that $b_W > 55.23\%$.

- b) [6 pts] Find a test statistic. Show the formula that you use, not just the numerical answer. Let B denote the actual bounce rate observed in $n = 1792$ visits on weekends. Then under H_0 ,

$$Z = \frac{B - 55.23\%}{\sqrt{55.23\%(1 - 55.23\%)/n}}$$

has approximately a $N(0, 1)$ distribution. In our case

$$\frac{.5894 - .5523}{\sqrt{.5523(1 - .5523)/1792}} = \frac{.0371}{\sqrt{.5523(.4477)/1792}} = \frac{.0371}{0.0117465969} = 3.15836155,$$

so the value of this statistic is $Z = 3.16$.

- c) [6 pts] Find the p-value for your test.

From the table, $\mathbb{P}(Z \leq 3.16) = 99.92\%$, so the p-value is $0.08\% = .0008$.

- d) [6 pts] Write a sentence or two explaining the meaning of the p-value you calculated in part c, in the context of this problem. (If you got stuck before part c, you may pretend the p-value is .08 and use this number for parts d, e, and f.)

Under the null hypothesis, the probability of observing such a high bounce rate on some weekends as we did is only a small fraction of a percent, namely 0.08%.

- e) [5 pts] Are you able to reject the null hypothesis (use significance level .05)?

Since $0.08 < .05$, we are indeed able to reject H_0 .

- f) [5 pts] Write a sentence explaining your conclusion in the context of the problem.

There is very strong evidence that the bounce rate is higher on weekends.

Score: _____

Additional space for answers to Problem 3.

Score: _____

Problem 4. Suppose the number of hours past noon that an old alarm clock goes off has a density given by

$$f(x) = \begin{cases} \frac{2x}{9} & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

(a) [7 points] What is the probability that the alarm clock goes off between 2pm and 3pm?

Solution: $\int_2^3 \frac{2x}{9} dx = \frac{2}{9} \left(\frac{3^2}{2} - \frac{2^2}{2} \right) = \frac{5}{9}$.

(b) [7 points] What is the probability that the alarm clock goes off between noon and 1pm?

Solution: $\int_0^1 \frac{2x}{9} dx = \frac{2}{9} \frac{1^2}{2} = \frac{1}{9}$.

Score: _____

Problem	Score
1	/19
2	/18
3	/34
4	/14
5	/15
Total	/100

Problem 5. [15 pts.] Suppose the number of *new* visitors to the web site *math.hawaii.edu* during a month is normally distributed with a mean of 3,389 and a standard deviation of 1,000, and the number of *returning* visitors is normally distributed with a mean of 6,419 and a standard deviation of 2,000. Assume that the numbers of new and returning visitors are independent of one another. Find the probability that there will be the more new visitors than returning visitors during a given month.

Solution: Let N and R be the number of New and Returning visitors, respectively. We have $\mu_N = 3389$, $\sigma_N = 1000$, $\mu_R = 6419$, and $\sigma_R = 2000$. Letting $X = N - R$, we seek $\mathbb{P}(N > R) = \mathbb{P}(N - R > 0) = \mathbb{P}(X > 0)$. The random variable X has

$$\mu_X = \mu_N - \mu_R = 3389 - 6419 = -3030,$$

$$\text{Var}(X) = \text{Var}(N - R) = \text{Var}(N) + (-1)^2 \text{Var}(R) = 1000^2 + 2000^2 = 5 \cdot 10^6$$

and

$$\sigma_X = \sqrt{\text{Var}(X)} = \sqrt{5} \cdot 10^3 = 2236.1$$

So

$$\begin{aligned} \mathbb{P}(X > 0) &= \mathbb{P}\left(\frac{X - (-3030)}{2236.1} > \frac{-(-3030)}{2236.1} = 1.36\right) \approx \mathbb{P}(Z > 1.36) \\ &= 1 - \mathbb{P}(Z < 1.36) = 1 - 91.31\% = 8.69\%. \end{aligned}$$

where $Z \sim N(0, 1)$.

Useful Formulas

	Discrete case	Continuous case
$P(a \leq X \leq b)$	$\sum_{i:a \leq x_i \leq b} P(X = x_i)$	$\int_a^b f(x) dx$
$\mu = E[X]$	$\sum_i x_i P(X = x_i)$	$\int_{-\infty}^{\infty} x f(x) dx$
$\text{Var}(X)$	$\sum_i (x_i - \mu)^2 P(X = x_i)$	$\left(\int_{-\infty}^{\infty} x^2 f(x) dx \right) - \mu^2$
$\text{SD}(X)$	$\sqrt{\text{Var}(X)}$	$\sqrt{\text{Var}(X)}$

Distribution	probability function or density	E[X]	SD(X)
Binomial(n, p)	$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k = 0, 1, \dots, n$	np	$\sqrt{np(1-p)}$
Poisson(λ)	$P(X = k) = e^{-\lambda} \lambda^k / k!$ for $k = 0, 1, \dots$	λ	$\sqrt{\lambda}$
Geometric(p)	$P(X = k) = (1-p)^{k-1} p$ for $k = 1, 2, \dots$	$1/p$	$\sqrt{1-p}/p$
Uniform(a, b)	$f(x) = 1/(b-a)$ for $a < x < b$	$(a+b)/2$	$(b-a)/\sqrt{12}$
Exponential(λ)	$f(x) = \lambda e^{-\lambda x}$ for $x > 0$	$1/\lambda$	$1/\lambda$
Normal(μ, σ^2)	$f(x) = (2\pi\sigma^2)^{-1/2} e^{-(x-\mu)^2/2\sigma^2}$	μ	σ

$$E[X + c] = E[X] + c \qquad E[cX] = cE[X] \qquad E[X + Y] = E[X] + E[Y].$$

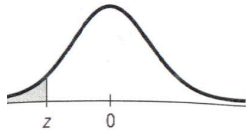
$$\text{Var}(X + c) = \text{Var}(X) \qquad \text{Var}(cX) = c^2 \text{Var}(X)$$

$\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ if X and Y are independent.

$$E[\bar{X}] = \mu \qquad \text{SD}(\bar{X}) = \sigma/\sqrt{n} \qquad E[\hat{p}] = p \qquad \text{SD}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

$$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \qquad Z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

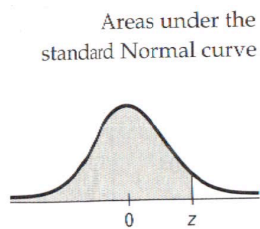
Areas under the standard Normal curve



0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000 [†] -3.9
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.8
0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	-3.7
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.6
0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	-3.5
0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	-3.4
0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	-3.3
0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007	-3.2
0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010	-3.1
0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013	-3.0
0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019	-2.9
0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026	-2.8
0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035	-2.7
0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047	-2.6
0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062	-2.5
0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082	-2.4
0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107	-2.3
0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139	-2.2
0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179	-2.1
0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228	-2.0
0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287	-1.9
0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359	-1.8
0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446	-1.7
0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548	-1.6
0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668	-1.5
0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808	-1.4
0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968	-1.3
0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151	-1.2
0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357	-1.1
0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587	-1.0
0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841	-0.9
0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119	-0.8
0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420	-0.7
0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743	-0.6
0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085	-0.5
0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446	-0.4
0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821	-0.3
0.3859	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207	-0.2
0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602	-0.1
0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000	-0.0

[†]For $z \leq -3.90$, the areas are 0.0000 to four decimal places.

Figure 1: Areas under the standard normal curve.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000 [†]									

[†]For $z \geq 3.90$, the areas are 1.0000 to four decimal places.

Figure 2: Areas under the standard normal curve, continued.