Elementary Probability and Statistics
Midterm II
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Disclaimer: It is essential to write legibly and show your work. If your work is absent or illegible, and at the same time your answer is not perfectly correct, then no partial credit can be awarded. Completely correct answers which are given without justification may receive little or no credit.

During this exam, you are not permitted to use notes, or books, nor to collaborate with others. You are allowed to use a calculator.
Problem 1. Suppose students enter the University of Phoenix psychology PhD program at a rate of 10 per year on average. Among these students, 60% are women.

(a) (7 points) What is the probability that exactly 4 out of the next 10 students entering the program are women?

(b) (6 points) What is the probability that exactly 9 students enter the program in the next two years?

(c) (6 points) What is the probability that it will be between 1 and 2 months before the next student enters the program? (Assume that students can enter the program at any time, i.e., not necessarily at the beginning of a term.)
Problem 2. [18 points] For each of the random variables below, indicate whether the distribution would be best described as binomial, geometric, Poisson, exponential, uniform, or normal. You may use an answer more than once.

a) The number of papers, out of the next 10 published by Professor Li, that have an even number (2, 4, 6, 8, etc.) of occurrences of the word “and”.

b) The number of emails that a person receives in a day.

c) The proportion who vote for Obama among 100 randomly selected voters in the U.S.

d) The number of times you have to toss a coin before getting a tail.

e) The amount of time before the next paper is published by Professor Li.

f) The sum of the weights of 10,000 randomly chosen babies.
Problem 3.  \textit{Bounce rate} is an Internet marketing term used in web traffic analysis. It represents the percentage of visitors who enter the site and “bounce” (leave the site) rather than continue viewing other pages within the same site. Suppose that for the web site \texttt{math.hawaii.edu}, the overall bounce rate is 55.23\% on average.

On some recent weekend days the average bounce rate was observed to be 58.94\%. The total number of visitors on the observed days was 1,792. Do these results provide strong evidence that the bounce rate is higher on weekends?

a) [6 pts] State appropriate null and alternative hypotheses.

b) [6 pts] Find a test statistic. Show the formula that you use, not just the numerical answer.

c) [6 pts] Find the p-value for your test.

d) [6 pts] Write a sentence or two explaining the meaning of the p-value you calculated in part c, in the context of this problem. (If you got stuck before part c, you may pretend the p-value is .08 and use this number for parts d, e, and f.)

e) [5 pts] Are you able to reject the null hypothesis (use significance level .05)?

f) [5 pts] Write a sentence explaining your conclusion in the context of the problem.
Score: ______

Additional space for answers to Problem 3.
Problem 4. Suppose the number of hours past noon that an old alarm clock goes off has a density given by

\[ f(x) = \begin{cases} \frac{2x}{9} & \text{if } 0 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases} \]

(a) [7 points] What is the probability that the alarm clock goes off between 2pm and 3pm?

(b) [7 points] What is the probability that the alarm clock goes off between noon and 1pm?
Problem 5. [15 pts.] Suppose the number of new visitors to the web site \( \text{math.hawaii.edu} \) during a month is normally distributed with a mean of 3,389 and a standard deviation of 1,000, and the number of returning visitors is normally distributed with a mean of 6,419 and a standard deviation of 2,000. Assume that the numbers of new and returning visitors are independent of one another. Find the probability that there will be the more new visitors than returning visitors during a given month.
Useful Formulas

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability function or density</th>
<th>E[X]</th>
<th>SD(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binomial(n, p)</td>
<td>( P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} ) for ( k = 0, 1, ..., n )</td>
<td>( np )</td>
<td>( \sqrt{np(1-p)} )</td>
</tr>
<tr>
<td>Poisson(( \lambda ))</td>
<td>( P(X = k) = e^{-\lambda} \lambda^k / k! ) for ( k = 0, 1, ... )</td>
<td>( \lambda )</td>
<td>( \sqrt{\lambda} )</td>
</tr>
<tr>
<td>Geometric(p)</td>
<td>( P(X = k) = (1-p)^{k-1} p ) for ( k = 1, 2, ... )</td>
<td>( 1/p )</td>
<td>( \sqrt{\frac{1-p}{p}} )</td>
</tr>
<tr>
<td>Uniform(a, b)</td>
<td>( f(x) = 1/(b-a) ) for ( a &lt; x &lt; b )</td>
<td>( (a+b)/2 )</td>
<td>( (b-a)/\sqrt{12} )</td>
</tr>
<tr>
<td>Exponential(( \lambda ))</td>
<td>( f(x) = \lambda e^{-\lambda x} ) for ( x &gt; 0 )</td>
<td>( 1/\lambda )</td>
<td>( 1/\lambda )</td>
</tr>
<tr>
<td>Normal(( \mu, \sigma^2 ))</td>
<td>( f(x) = (2\pi\sigma^2)^{-1/2} e^{-(x-\mu)^2/2\sigma^2} )</td>
<td>( \mu )</td>
<td>( \sigma )</td>
</tr>
</tbody>
</table>

\[ E[X + c] = E[X] + c \quad E[cX] = cE[X] \quad E[X + Y] = E[X] + E[Y]. \]

\[ \text{Var}(X + c) = \text{Var}(X) \quad \text{Var}(cX) = c^2 \text{Var}(X) \]

\[ \text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) \text{ if } X \text{ and } Y \text{ are independent.} \]

\[ E[\bar{X}] = \mu \quad SD(\bar{X}) = \sigma/\sqrt{n} \quad E[\hat{p}] = p \quad SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}} \]

\[ ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad Z = \frac{p - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \]
Figure 1: Areas under the standard normal curve.
Figure 2: Areas under the standard normal curve, continued.