Whale population: Consider the survival of a population of whales, and assume that if the number of whales falls below a minimum survival level $m$, then the species will become extinct. In addition, assume that the population is limited by the carrying capacity $M$ of the environment. That is, if the whale population is above $M$, then it will experience a decline because the environment cannot sustain that large a population level.

a) Let $a_n$ represent the whale population after $n$ years. Discuss the model

$$a_{n+1} = a_n + k(M - a_n)(a_n - m),$$

where $k > 0$. Does it make sense in terms of the description above?

b) Find the fixed points of the model, and determine their stability via linearization. You may assume that $M = 5000$, $m = 100$, and $k = 0.0001$.

c) Perform a graphical stability analysis. Are you results consistent with the results from b)?

d) Sketch the graphs of $a_n$ versus $n$ for three initial conditions. Try to pick initial conditions that show different behaviors.

e) The model has two serious shortcomings. What are they? Hint: Consider what happens when $a_0 < m$, and when $a_0 >> M$.

f) Extra credit: Think of a possible way to fix the model so as to overcome the shortcomings. You are encouraged to be creative, innovative - you do not need to write down the equation of the improved model; it is sufficient to describe your ideas with words and/or sketches of graphs.