Let $x = \text{the increase in room rate.}$

Then the revenue is

$$R(x) = (\text{room rate}) (\text{number of rooms occupied}) = (80 + x)(300 - x) = 24000 + 220x - x^2$$

Since the decrease in the number of rooms occupied is equal to $x$.

The cost is $C(x) = 22(\text{number of rooms occupied})$

$$= 22(300 - x)$$

$$= 6600 - 22x$$

The profit is $P(x) = R(x) - C(x)$

$$= 24000 + 220x - x^2 - (6600 - 22x)$$

$$= 17400 + 242x - x^2$$

Find $x$ that maximizes profit:

$$P'(x) = 242 - 2x$$

so $P'(x) = 0 \implies 2x = 242 \iff x = 121$

$$P''(x) = -2$$

so $P''(121) = -2 < 0$. Thus $x = 121$ is a relative maximum and since $P''(x) < 0$ for all $x$, it is the absolute maximum.

Therefore the room rate should be $80 + 121 = 201$.

In order to maximize profits,