

Math 242 Lab (Feb 08, 2008)

1 Reminder

From the next week on

1. there will be 1~3 problems in each executed lab session that are required to be solved and whose answers are required to be turned in at the end of that session.
2. there will be a quiz of at most 3 problems every 2 weeks

2 Contents Covered

1. L'Hospital Rule
2. Infinitesimals (Supplementary)
3. Hyperbolic functions
4. Taylor Expansion (Supplementary)

3 Infinitesimals

Let's start with a simple example, say, x . It should be noted that x here is regarded as variable (ranging over all admissible values). We know that

$$\lim_{x \rightarrow 0} x = 0$$

Also, we know that

$$\lim_{x \rightarrow 0} \sin x = 0$$

We call $x, \sin x$ *infinitesimals*. Intuitively, any quantity that tends to zero is called an infinitesimal.

One natural question one tends to ask is that,

Amongst $x, \sin x$, which goes to zero faster as x tends to 0?

This is equivalent to solve

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = ?$$

which may take a while for the exact answer to appear.

4 Practice

- Graph $y = \tanh x = \frac{\sinh x}{\cosh x}$ and find out $\lim_{x \rightarrow +\infty} \tanh x$, $\lim_{x \rightarrow -\infty} \tanh x$ and $\lim_{x \rightarrow 0} \tanh x$. Find out its asymptotes. (Hint: graph the auxiliary functions $y = 1, y = -1$)
- Given the following pair of functions $f(x) = \tan x$ and $g(x) = \sin x$. Graph $\frac{f(x)}{g(x)}, \frac{f'(x)}{g'(x)}$. What can you say about the relation between $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$ and $\frac{f'(0)}{g'(0)}$

- Given

$$f(x) = x \arctan \frac{1}{x}$$

for $x \neq 0$. (a) Is f continuous at 0? (b) Is f differentiable at 0?

Run MatLab. To answer (a), input the following commands

```
syms x                                % defines the symbol x
y=x*atan(1/x)                          % atan is arctan
limit(y,x,0,'right')                   %compute lim_{x→0+} y
limit(y,x,0,'left')                    %compute lim_{x→0-} y
```

Now, can you give the answer to (a)? (Write your answer down)

Remark: the percentage sign and the words following it are for comments, not to be inputed as commands.

To answer (b), input the following commands

```
diff(y,x,1)                            % compute the 1st order
derivative of y w.r.t x
```

(not a command) highlight and copy the function you got

```
z=atan(1/x)-1/x/(1+1/x^2)               % denote the dy/dx by z
limit(z,x,0,'left')                     % compute lim_{x→0-} z
limit(z,x,0,'right')                    % compute lim_{x→0+} z
```

Now, can you give the answer to (b)? (Write your answer down)

Finally, to verify your answers again, input the following commands

```
x=-2:0.05:4;                            % define an array w.r.t. x
values
y=x.*atan(1./x);                         % define y = x arctan(1/x)
```

