

Math 242 Lab (Jan 25, 2008)

Please turn in last week's homework assignments, thanks.

Today MatLab will be used to aid your math study and I hope you'll enjoy it. :)

1. Topics Covered

- 1.1 Indefinite Integral
- 1.2 Convergence of Sequence
- 1.3 Newton's Method

2. Indefinite integral

Suppose $f(x)$ is integrable on some interval $[a, b]$, then for any $x \in [a, b]$, the function

$$g(x) = \int_a^x f(t)dt$$

is called the **Indefinite Integral** of f

Remark: Different definitions of Indefinite Integral may be given, but this is the easiest one. Also, the interval $[a, b]$ may be $(-\infty, b]$ or $(a, +\infty)$ or $(-\infty, +\infty)$.

3. Convergence of Sequence

Suppose $\{x_n\}_{n \in \mathbb{N}}$ is an infinite sequence of real numbers, if there is a finite real number α such that

For any $\varepsilon > 0$, there exists some $N \in \mathbb{N}$ such that $|x_n - \alpha| < \varepsilon$ for all $n > N$

then α is called the limit of the sequence, or, written as

$$\lim_{n \rightarrow \infty} x_n = \alpha$$

Remark: The definition of $\lim_{n \rightarrow \infty} x_n = \infty$ is not given here but it should be obvious.

4. Newton's Method

Newton's method is a powerful numerical tool to find the roots of an equation by **iterations**. To apply the Newton's Method to a function, the following conditions must be satisfied

1. f is differentiable on some interval (a, b)
2. there are x_1, x_2 with $a < x_1 < x_2 < b$ such that

$$f(x_1)f(x_2) < 0$$

Remark: In this interval (a, b) , it may happen that f has more than one root. The following additional condition insures that f has one and only one root in $[x_1, x_2]$

3. f' maintains a fixed sign, i.e., $f'(x) > 0$ or $f'(x) < 0$ for $x \in [x_1, x_2]$

The basic step of Newton's Method are

a) pick points x_1, x_2 , say, $x_1 < x_2$ with $f(x_1)f(x_2) < 0$ and $f'(x) > 0$ or $f'(x) < 0$ for $x \in [x_1, x_2]$

b) choose any $x_0 \in (x_1, x_2)$ as an initial value, apply the iteration equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

until the given precision is achieved.

4. Problems

4.1 Show that

$$\sum_{k=1}^n \frac{1}{k} < \ln n < \sum_{k=1}^{n-1} \frac{1}{k}$$

Hint: use the definition

$$\ln x = \int_1^x \frac{1}{t} dt$$

if you want a pure mathematical proof. Else, run MatLab and input the following codes

```
n=1:1:1000;
sum=1;
for n=1:1000
sum=sum+1/n;
end
sum
```

to get an approximation of $\sum_{k=1}^n$ for $n = 1000$; input

$$\log(1000)$$

to get the value of $\ln(1000)$, and input

```

n=1:1:999;
sum=1;
for n=1:999
sum=sum+1/n;
end
sum

```

to get $\sum_{k=1}^{n-1} \frac{1}{k}$ also for $n = 1000$.

Caution: Colons in the commands are NEEDED

4.2. Give a brief explanation on why

$$e^x + x = 0$$

has **one and only one root**. Use the graph of the function

$$f(x) = e^x + x \tag{1}$$

to find the approximate root of (1).

Step 1: To plot the graph of $f(x)$ by MatLab, input the following codes

```

x=-3:0.1:1;
y=exp(x)+x;
plot(x,y)

```

Step 2: To find the roots, click "File", then "New", then "M-File" and in the popup edit window, input the following commands:

```

function y=g(x)
y=exp(x)+x;

```

When finished, save the file with the default name g.m.

Now, in MatLab Command window input:

```

x1=fzero('g',0.5)
x1 = -0.5671

```

where in first line above, "**0.5**" is the initial value which can be set differently, but, to be within the interval where the root exists.

Step 3: To carry out the iteration process. Create another mfile by inputting the following:

```
function z=h(x);
z=exp(x)+1;
```

and save the file with the default name h.m.

Step 4: In Matlab Command Window, input the following commands:

```
format long
n=1:1:1000;
x=0.5;
for n=1:1000;
x=x-(g(x)/h(x));
end
x
```

This will iterate the steps in Newton's Method for a 1000 loops and give the answer, which is

$$x = -0.56714329040978$$

4.3 Graph

$$p(t) = \frac{1}{1 + ae^{-kt}}$$

for $a = 10$, $k = 0.5$, where

- a and k are constants and t is taken as the variable
- p is the proportion of the population knowing the rumor at time t with the given a, k

Estimate: How long will it take for 80% of the population to hear the rumor for a, k given above.

(Easy: use Derive)