Math 252 (April 9, 2008)

1 Reminders

Please write your answers on the space given hereunder or on your own sheets if not enough space is available. Thanks

2 Summary

The process of solving a differential equation (DE) is the reverse operation of differentiation and most of the hard DE's only permits numerical solutions. Today's content will cover

1. Basic techniques of solving 1st Order Linear Ordinary DE's,
2. MatLab Commands for solving these equations

3 Backgrounds

Let's start from a very simple but important example. If

$$f(x) = e^x + c \tag{1}$$

where $c \in \mathbb{R}$ is an arbitrary constant, then

$$f^{(n)}(x) = \frac{d^{(n)}}{dx^{(n)}} f(x) \equiv e^x \tag{2}$$

for all $n \in \mathbb{N}^+ = \{1, 2, 3, ...\}$.

Reversing (2) is equivalent to solving the simplest ODE and $f(x)$ in (1) is the only (family) (Can you tell what "family" means here?) of non-constant functions satisfying (2). It is these two magic equations that make solving certain types of ODE's or even Partial DE's a lot easier and systematic.

4 First Order Linear DE's

Separable ODE's and general first order linear DE's will be treated together as

$$\frac{dy}{dx} + P(x)\, y = Q(x) \tag{3}$$

where $P, Q \in C^0([a, b])$ and the *integrating factor* method will be used to solve them.

Before coming to this method, please solve

$$\frac{d}{dx} e^{\int f(t)dt} = ?$$

The essense of integrating factor method (which is given on Page 536 of the text book) is to create an *exact differential* on the left-side of the equation (in the present setting). By multiplying

$$I(x) = e^{\int P(t)dt} \tag{4}$$

on both sides of (3), it's obtained that

$$d(I(x)y) = (I(x)Q(x))dx \tag{5}$$

Thus the given equation is solved. Note here (3) is written slightly differently from the one given in the textbook in that, the left-side here is an *exact differential*

5 Practice

Problem 1 *Pleae try to solve*

$$\frac{dy}{dx} + 2xy = x^2$$

via pure mathematical reasoning, and, then execute these commands in MatLab:

```
>dsolve('Dy=1/(x^2-2*x*y)','x')
```

Is your answer correct?