Math 251 (Jan 30, 2008)

Topics Covered:
1. Rate of Increasing
2. Harmonic sequece
3. Iterated Algorithm (To be introduced next time)

Problems

1. With computer aided reasoning, please try your best to prove the following assertion

$$\lim_{x\to\infty} \frac{x^n}{e^x} = 0 \qquad (1)$$

for any **prespecified** $n \in \mathbb{N}$.

Hint: Try to graph $y = e^x$ and $y = x^n$ in the same window using whatever math software you like and find out how fast these functions are increasing.

2. With the assertion given in (1), can you make an educated guess of the limit of

$$\lim_{x\to\infty} \frac{x^m}{\ln x} = \infty$$

for any prespecified $m \in \mathbb{N}$

Hint: What's the relation between $y = \ln x$ and $y = e^x$.

3. What can you say about the relative rates of increasing of exponential, power, logarithm functions?

(Try your best to answer this question correctly)

4. Show that

$$\sum_{k=2}^{n-1} \frac{1}{k} < \ln n < \sum_{k=1}^{n} \frac{1}{k}$$

Hint: use the definition

$$\ln x = \int_1^x \frac{1}{t} dt$$

if you want a pure mathematical proof. Else, let's run MatLab to estimate the sum with $n = 2008$. Click on MatLab icon on the programs list and in the command window of MatLab, input the following codes to estimate $\sum_{k=1}^{2008} \frac{1}{k}$:

```
format long
n=1:1:2008;
sum=1;
for n=1:2008
sum=sum+1/n;
end
sum
```

To estimate $\sum_{k=2}^{2008-1} \frac{1}{k}$, just input these following codes:

```
format long
n=2:1:2007;
sum=1/2;
for n=1:2007
sum=sum+1/n;
end
sum
```

Finally, to get $\log(2008)$,just input:

```
ln(2008)
```

Now let

$$s_n = \sum_{k=1}^{n} \frac{1}{k}$$

What can you say about

$$\lim_{n\to\infty} s_n$$

and

$$\lim_{x\to\infty} \ln(x)$$

and

$$\lim_{n\to\infty} \frac{\ln(n)}{s_n} \tag{2}$$

Remark: Try your best to answer (2).

5. Can you guess what function $y = f(x)$ satisfy the following equation

$$\frac{dy}{dx} = kxy \tag{3}$$

If so, give the explicit solution of (3); if not, make a guess on how fast the solution $y = f(x)$ is growing compared to power functions.