Math 252 Lab (Feb 20,2008)

Today's assignments may be a bit harder and you can request to turn your answers in next week if need be. Please try your best to do these problems and extra credits will be given for excellent performance.

1 Contents Covered

1. $\varepsilon - \delta$ argument

   If $\{a_n\}$ is sequence, then

   $$\lim_{n\to\infty} a_n = L$$

   means that for every $\varepsilon > 0$ there is a corresponding integer N such that

   $$|a_n - L| < \varepsilon \quad \text{whenever} \;\; n > N$$

2. Criteria on convergence

   (a) Every bounded, monotonic sequence is convergent.

   (b) Every Cauchy sequence is convergent. (Supplementary)

3. Imporper integral

   Definite integrals where the interval of integration is unbounded or the integrand is unbounded are called improper integrals, say,

   $$\int_1^\infty \frac{dx}{x^2}, \qquad \int_0^1 \frac{dx}{\sqrt{x}}, \qquad \int_{-1}^1 \frac{dx}{\sqrt{x}}$$

   Generally, for each such integral of the type

   $$\int_a^b f(x)\,dx$$

   (a) If either $b = \infty$ or/and $a = -\infty$, but f is bounded on the interval (a, b) then

   $$\int_a^b f(x)\,dx := \lim_{d\to b, c\to a} \int_c^d f(x)\,dx$$

   (if the limit on the rightside exists.)

(b) If a, b are both finite but f is not bounded at some point $x_0 \in [a, b]$, then

$$\int_a^b f(x)\,dx := \lim_{d \to 0+} \int_{x_0+d}^b f(x)\,dx + \lim_{c \to 0-} \int_{x_0-c}^b f(x)\,dx$$

(if the limit on the rightside exists.)

Remark 1 *It may happen that both the interval of integration and the integrand are unbounded*

Remark 2 *There is a strong connection between imporper integrals and series*

2 Practice

1. Use Derive to find $\lim_{n\to\infty} r^n$ (Note that $|r|$ is not only restricted to be less than 1)

   Question: In which case is the sequence $\{r^n\}$ monotonic?

2. Use Derive to find $\int_1^\infty \frac{dx}{x^2}, \int_0^1 \frac{dx}{\sqrt{x}}$

   Assignment: Please use the definition of improper intergal to find the answer to $\int_0^1 \frac{dx}{\sqrt{x}}$ and turn it in.

3. Use Derive to find $\sum_{n=1}^\infty \frac{1}{n^2}$. (Hint: Define $s_n = \sum_{k=1}^n \frac{1}{k^2}$ and then use Derive to find the limit of the sequence $\{s_n\}$)

   Assignment: Please use one of the criteria to show the sequence $\{s_n\}$ converges. (Hints: To show that $\{s_n\}$ is bounded and monotonic.)

3 Assignment

1. Please give a **bounded**, **divergent** sequence.