Math 242 Lab (Feb 15, 2008)

1 Contents

1. Differentiation
2. Integration by parts
3. Trigonometric functions

2 Practice Problems

Since Derive is NOT able to do integration by parts, let's do some manual computation and then use Derive to check the answers.

1. Show that $\int x^n e^x dx = x^n e^x - n\int x^n e^x dx$ for $n \in \mathbb{N}$ using integration by parts.
2. Solve $\int \frac{dt}{\sin t + \cos t}$ (hint: let $x = \tan \frac{t}{2}$).

3 Assignments

Do Prob 1 and any one of 2 and 3. You can use Derive to get the answers and then manually compute this integrals. Provide your arguments, answers carefully and turn them in at the end of the session. Thanks.

1. Solve $\int x^3 \arctan x dx$
2. Use trigonometric substitution to show that

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right) + C$$

3. Or, use hyperbolic substituion $x = a \sin t$ to show that

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

4 Answer to Quiz Prb.

Original Problem:

Use MatLab to show that

$$f(x) = \begin{cases} |x|^x & if \quad x \neq 0 \\ 1 & if \quad x = 0 \end{cases}$$

is continuous at $x = 0$ with the help of graph and l'Hospital Rule. (Extra credit: Is f differentiable at $x = 0$?)

Solution: 1. By definition a function f is said to be continuous at $x = x_0$ if f is defined in some neighborhood of $x = x_0$ and $\lim_{x \to x_0 +} f(x) = f(x_0) = \lim_{x \to x_0 -} f(x) = \alpha$ where α is finite. So, it suffices to show

$$\lim_{x \to 0+} |x|^x = f(0) = 1$$

and

$$\lim_{x \to 0-} |x|^x = f(0) = 1$$

which is justified by Derive (or by applying L'Hospital's Rule to the auxiliary function $g(x) = x \ln |x|$ since $f(x) = |x|^x = \left(e^{\ln|x|}\right)^x$ and find out $\lim_{x \to 0} x \ln |x|$).

2. To show that f is NOT differentiable at $x = 0$. Just follow the definition to check (by Derive or by applying L'Hospital's Rule)

$$\lim_{x \to 0+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0+} \frac{|x|^x - 1}{x}$$

and

$$\lim_{x \to 0-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0-} \frac{|x|^x - 1}{x}$$

which are both $-\infty$ and justify the assertion.

It is true that if $f'(x)$ exists for $x \neq x_0$ but either or both of $\lim_{x \to x_0 +} f'(x)$ and $\lim_{x \to x_0 -} f'(x)$ is ∞ (or, $-\infty$) implies f is NOT differentiable at $x = x_0$.