**Math 140  Lecture 5**

*Exam 1, next week, Lectures 1-6*

**Basic graphs**
Know these graphs. Note: tangent to $x^3$ at the origin is horizontal.

- $x^2$,
- $x^3$,
- $1/x$,
- $\sqrt{x}$,
- $|x|$,
- $\sqrt{1-x^2}$.

**Translations and reflections**

**THEOREM.** Changing the value $f(x)$ changes the vertical position; changing the argument $x$ changes the horizontal position in *opposite the expected direction.*

<table>
<thead>
<tr>
<th>up 1 unit</th>
<th>down 1</th>
<th>*left 1</th>
<th>*right 1</th>
<th>reflect in x-axis</th>
<th>reflect in y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)+1$</td>
<td>$f(x)-1$</td>
<td>$f(x)+1$</td>
<td>$f(x)-1$</td>
<td>$-f(x)$</td>
<td>$f(-x)$</td>
</tr>
</tbody>
</table>

- Given $f(x)$, find the functions for the other graphs.

**VERTICAL MOVES WITH VALUE CHANGES**

- Given $f(x) = |x|$, graph $f(x)+2$, $f(x)-2$, $-f(x)$.
  $f(x) = |x|$, $f(x)+2 = |x|+2$, $f(x)-2 = |x|-2$, $-f(x) = -|x|$.

**HORIZONTAL MOVES WITH ARGUMENT CHANGES**

$x$ = the $x$-axis position. Changing $x$, changes the horizontal position of the coordinate system. Replacing $x$ by $x+2$ shifts the coordinate system 2 units to the left.

For a formula with several shifts and reflections of a function $f$, rewrite it in the *graph-translation form*

$$ af(b(x - c)) + d $$

Then the shifts and reflections occur in the *left-to-right order:*

- A negative $a$ gives a vertical reflection.
- A negative $b$ gives a horizontal reflection.
- The horizontal shift is determined by the $c$: right if $c$ is positive, left if $c$ is negative.
- The vertical shift is determined by $d$: up if $d$ is positive, down if $d$ is negative.

- $f(x) = 1/x$. Describe the following shifts.
  $f(x)+1 = (1/x)+1$ Shift up 1
  $f(x)-1 = -1/x$ Reflect in x-axis
  $f(x)+1 = -1/(x+1)$ Up 1, reflect in x-axis
  $f(x-1) = 1/(x-1)$ Shift right 1
  $f(x+1) = 1/(x+1)$ Shift left 1
  $f(x) = 1/x$ Reflect in y-axis
  $f(1-x) = 1/(1-x)$ Rewrite: $f(1-x)=-f(x+1)=-f(-x-1)$ Reflect in y-axis, shift right 1
  $f(-x+1) = 1/(-x+1)$ Rewrite: $f(-x-1)=-f(-x-1)$ Reflect in y-axis, shift right 1

- $1 - f(-x-1)$ Rewrite as

  $1 - f(-x-1) = -f((x+1)) + 1 = -f((x-(1))) + 1$

  Reflect in x-axis, reflect in y-axis, left 1, up 1.

- $g(x) = x^2 - x$. Graph Parabola with roots 0, 1
  $g(1+x)$ Graph Shift left 1
  $g(1-x)$ Graph Reflect in y-axis, shift right 1.