Math 140  Lecture 16
Fri. = last day to withdraw. Keller 419A secretary will sign for me.

**FACTS.** Know the sin, cos and tan of: 0, π/6, π/4, π/3, π/2.

\[ \sin(0) = 0 \quad \cos(0) = 1 \quad \tan(0) = 0 \]
\[ \sin(\pi/6) = 1/2 \quad \cos(\pi/6) = \sqrt{3}/2 \quad \tan(\pi/6) = 1/\sqrt{3} \]
\[ \sin(\pi/3) = \sqrt{3}/2 \quad \cos(\pi/3) = 1/2 \quad \tan(\pi/3) = \sqrt{3} \]
\[ \sin(\pi/2) = 1 \quad \cos(\pi/2) = 0 \quad \tan(\pi/2) = \text{undefined} \]

\* \( \sqrt{2}/2 \) is also ok.

**THEOREM.** The sin and cos of \( \theta \) equals the sin and cos of its reference angle except for the sign which is determined by \( \theta \)'s quadrant.

- List three angles (in radian measure) whose cos is ½.
  \( \pi/3, \pi/2, 5\pi/3 \)

**NOTATION.** sin\( \theta \) means sin(\( \theta \)); sin\(^2\)\( \theta \) means (sin(\( \theta \)))\(^2\).

**THEOREM.** Since sin\( \theta \) and cos\( \theta \) are the legs of a right triangle of hypotenuse 1,
\[ \text{sin}^2 \theta + \text{cos}^2 \theta = 1 \]

- \( \sin\theta = -2/3 \) and \( \pi < \theta < 3\pi/2 \). Find \( \cos \theta \) and tan\( \theta \).
  \((-2/3)^2 + \cos^2 \theta = 1 \)
  \( \cos\theta = \pm \sqrt{5}/3 \)
  \( \pi < \theta < 3\pi/2 \) is in quadrant III, thus \( \cos\theta < 0 \):.
  \[ \cos\theta = -\sqrt{5}/3 \]
  \[ \tan\theta = \sin\theta/\cos\theta = (-2/3)/(-\sqrt{5}/3) = 2/\sqrt{5} \]

- \( \sec\theta = -3 \) and \( \sin \theta < 0 \). Find tan\( \theta \). First find sin, cos.
  \( \cos\theta = -1/3 \)
  \( \sin^2 \theta + (-1/3)^2 = 1 \)
  \( \sin\theta = 1-1/9 = 8/9 \)
  \( \sin\theta = \pm \sqrt{8}/3 \). Since sin \( \theta \) < 0,
  \( \sin\theta = -\sqrt{8}/3 = -2\sqrt{2}/3 \). : \( \tan\theta = 2\sqrt{2} \).

- Simplify \( \cos(\theta + \cot\theta)(\tan \theta + \sec\theta) \)
  \( = \cos\theta \tan\theta + \cos\theta \sec\theta + \cot\theta \tan\theta + \cot\theta \sec\theta \)
  \( = \cos\theta (\sin\theta + \cos\theta) + \cos\theta (\sin\theta + \cos\theta) + \cos\theta (\sin\theta \cos\theta + \cos\theta \sin\theta) \)
  \( = \sin\theta + 1 + 1 + \sin\theta \)
  \( = \sin\theta + 2 + \csc \theta \).

Note: \( y^2 + 3y + 2 \) factors into \((y+1)(y+2)\).

- Factor: \( \csc^2 \theta + 3\csc \theta + 2 = (\csc \theta + 1)(\csc \theta + 2) \).
Note: \( x^2 - y^2 = (x+y)(x-y) \).

- Factor: \( \sin^2 B - \cos^2 B = (\sin B - \cos B)(\sin B + \cos B) \).