Math 140 Lecture 19

Graphs of sin and cos

Recall. sin and cos have period $2\pi$:

\[
\sin(x + 2\pi) = \sin(x), \quad \cos(x + 2\pi) = \cos(x).
\]

We often graph periodic functions only over one period, e.g., $[0, 2\pi]$. Before and after this interval, they repeat.

Graph $\sin(x)$.

\[
\begin{align*}
\text{min} & \quad \text{root} & \quad \text{max} & \quad \text{root} & \quad \text{min} & \quad \text{root} \\
x\text{-intercept:} & \quad x = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots \\
\text{Max value:} & \quad 1 \text{ at } x = \ldots, -3\pi/2, \pi/2, 5\pi/2, \ldots \\
\text{Min value:} & \quad -1 \text{ at } x = \ldots, -5\pi/2, -\pi/2, 3\pi/2, \ldots \\
\text{Amplitude:} & \quad A = 1 \text{ (see definition below)} \\
\text{Period:} & \quad p = 2\pi
\end{align*}
\]

At $x = 0$, the line $y = x$ is tangent to the graph of $\sin(x)$.

Graph $\cos(x)$. ... Done similarly ...

Definition. The \textit{amplitude} of a function $f$ is half the difference between the max and min values of $f$. Like periods, amplitudes are always positive.

- Find the amplitude and period of $f$, $g$, and $h$.

- Graph $y = 2\sin(x)$ over one period.

- Graph $y = \sin(2x)$ over one period.

Note $\sin(x) = 0$ iff $x = \ldots, 0, \pi, 2\pi, \ldots$.

Thus $\sin(2x) = 0$ iff $2x = \ldots, 0, \pi, 2\pi, \ldots$ iff $x = 0, \pi/2, \pi, \ldots$.

Amplitude = 1, period = $\pi$.

Increases on $[0, \pi/4]$ and $[3\pi/4, \pi]$.

For $B > 0$, $y = \sin(Bx)$ has period $2\pi/B$.

Reason: $\sin(Bx)$ repeats at $Bx = 2\pi$. Solving for $x$ gives $x = 2\pi/B$.

$B$ is the compression factor.

Theorem. For $y = \pm A\sin(Bx)$ & $y = \pm A\cos(Bx)$ with $A, B > 0$,

- amplitude: $A$
- period: $p = 2\pi/B$. Hence also, $B = 2\pi/p$.

To graph, mark $\pm A$ on the y-axis, 0 on the x-axis.

Divide $[0, p]$ into four parts.

- Graph $y = -3\cos(\pi x)$ over one period.

List the amplitude, period, x-intercepts and the intervals in the period on which the function increases.

\[
y = -3\cos(\pi x) = -A\cos(Bx)
\]

$A = 3$, $B = \pi$

amplitude: $A = 3$

period: $p = 2\pi/B = 2\pi/\pi = 2$

Draw a one-period box of amplitude $A$ and length $p$.

Divide the period into four parts.

- Find an equation for the graph. Write it in the form $y = \pm A\sin(Bx)$ or $y = \pm A\cos(Bx)$ with $A, B > 0$.

The graph has the shape of $y = -A\cos(Bx)$.

amplitude: $A = 2$; period: $p = 4$.

$B$: $B = \frac{2\pi}{p} = \frac{2\pi}{4} = \frac{\pi}{2}$

Equation: $y = -2\cos\left(\frac{\pi}{2}x\right)$