Math 140 Lecture 29

- Find the focus, directrix and graph of \( y^2 + 2y = 4x - 5 \).
  \[ y^2 + 2y + 1 = 4x - 4, \quad (y + 1)^2 = 4(x - 1) \]
  \[ p = k/4 = 4/4 = 1. \]
  This is the parabola \( y^2 = 4x \) shifted: down 1 unit, right 1 unit.

- The \( y^2 \) means the parabola is horizontal.
  - Vertex: \( (0,0) \) shifted down 1 and right 1 \( \rightarrow (1,-1) \).
  - Focus: \( (p,0) = (1,0) \) shifted down 1, right 1 \( \rightarrow (2,-1) \).
  - Directrix: \( x = -p, i.e., x = -1 \), down 1, right 1 \( \rightarrow x = 0 \).

 **RECALL.** A circle is the set of all points such that the
distance to a center point is some constant \( r \).

**DEFINITION.** An ellipse is the set of all points such that
the sum of the distances to two focus points is the distance
between the vertices. The vertices are the two points
farthest apart. The major axis goes from a vertex at one
end of the ellipse through the two foci to the vertex at the
opposite end. The minor axis is a perpendicular bisector
of the major axis.

A light ray emitted from one focus point is reflected to
the opposite focus point. Planetary orbits are ellipses.

Let
\[ a = \text{major radius} = \frac{1}{2} \text{ the major axis length}. \]
\[ b = \text{minor radius} = \frac{1}{2} \text{ the minor axis length}. \]
\[ c = \text{focal radius} = \frac{1}{2} \text{ the distance between the foci}. \]

**THEOREM.** \( a^2 = b^2 + c^2 \). \( \therefore \ c = \sqrt{a^2 - b^2} \).

The graph of \( x^2 + y^2 = r^2 \) is a circle with center \( (0,0) \)
and radius \( r \). This can be written as \( \frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 \).

**THEOREM.** For \( a \geq b > 0 \), the graphs of \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) and
\( \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \) are ellipses. \( (0,0) \) is the center. \( a, b, c \) are the
major, minor and focal radii where \( c = \sqrt{a^2 - b^2} \).

\( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is a horizontal ellipse.
Foci: \( (-c,0) \) and \( (c,0) \).
Major axis: the line segment \( (-a,0)(a,0) \).
Minor axis: \( (0,-b)(0,b) \).

\( \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \) is a vertical ellipse.
Foci: \( (0,-c) \) and \( (0,c) \).
Major axis: \( (0,-a)(0,a) \).
Minor axis: \( (-b,0)(b,0) \).

To graph, complete the squares if some variable isn’t a
square. Get 1 on the right. Write the equation in one of
the forms above.

- Find the major and minor axes and foci; draw the graph:
  \( 4x^2 + y^2 = 4 \).
  \[ x^2 + \frac{y^2}{4} = 1, \quad \frac{x^2}{1^2} + \frac{y^2}{2^2} = 1, \quad \frac{y^2}{2^2} + \frac{x^2}{1^2} = 1 \]
  \( a = 2, b = 1, c = \sqrt{4-1} = \sqrt{3} \)

  Vertical ellipse \( y \) has the bigger radius.
  - Major axis: \( (0,-2)(0,2) \). Write your axes like this on the final.
  - Minor axis: \( (-1,0)(1,0) \).
  - Foci: \( (0,\sqrt{3}), (0,-\sqrt{3}) \) or \( (0,\pm \sqrt{3}) \).
  
  Set your compass to a
  - major radius.
  - Put the point at the end of a
  - minor radius.
  - Draw an arc.
  - It intersects the major axis
  - at the two foci.

- Find the major and minor axes and foci; draw the graph:
  \( 16x^2 - 96x + 25y^2 = 256 \).
  \[ 16(x^2 - 6x + \quad ) + 25y^2 = 256 + 16 \cdot 9 \]
  \[ 16(x-3)^2 + 25y^2 = 400 \]
  \[ (x-3)^2 + \frac{y^2}{16} = 1 \]
  \[ \frac{(x-3)^2}{5^2} + \frac{y^2}{4^2} = 1 \]
  \( a = 5, b = 4, c = \sqrt{25 - 16} = \sqrt{9} = 3 \)

  Horizontal ellipse \( x \) has the bigger radius.
  - Complete square for \( x \)
  - Divide by 400, get 1 on right
  - \( (x-3)^2 \)
  - Major axis: \( (-5,0)(5,0) \) shifted right 3 \( \rightarrow (2,0)(8,0) \).
  - Minor axis: \( (0,-4)(0,4) \) shifted right 3 \( \rightarrow (3,-4)(3,4) \).
  - Foci: \( (-3,0), (3,0) \) shifted right 3 \( \rightarrow (0,6), (0,6) \).