All exams will consist solely of homework type problems.

For odd degree vertical asymptotes, one side goes to $+\infty$, the other to $-\infty$.

For even degree vertical asymptotes, both sides go to $+\infty$ or both go to $-\infty$.

For a reduced rational function:
- $x$-intercepts (roots) occur where the top is 0.
- If the root has degree $n$, the $x$-intercept looks like that of $y = x^n$ or $y = x^{-n}$.
- If the bottom is 0 at $a$, then $x = a$ is a vertical asymptote.
- If the factor has degree $n$, the vertical asymptote looks like that of $y = 1/x^n$ or $y = -1/x^n$.

As $x \to \pm \infty$, the graph resembles the graph of the leading term which is either a constant $b$ or of the form $ax^n$ or $ax^{-n}$.

(1) If a constant $b$, then $y = b$ is a horizontal asymptote.
(2) If it is $ax^n$, then $y = 0$ is a horizontal asymptote.
(3) If it is $ax^{-n}$, there is no horizontal asymptote.

Definition. An exponential function is of the form $y = b^x$ with the base $b > 0$.

$b^0 = 1$, $b^1 = b$, $b^2 = b \cdot b$, ... $b^n = 1b^n$.

$b^{-n} = \sqrt[n]{b}$, the $n$th root of $b$.

$b^{mp} = b^{mp}$, $b^{km} = b^{km}$.

$\frac{b^m}{b^n} = b^{m-n}$.

$(ab)^n = a^n b^n$.

$(\frac{a}{b})^n = \frac{a^n}{b^n}$.

Property. If $b \neq 1$, $b^x = b^y \iff x = y$. Hence $b^x$ is 1-1.

$a \approx b$ means $a$ is approximately equal to $b$.

Fact: $2^{10} \approx 10^3$, i.e., $2^{10}$ is approximately equal to $10^3$.

The graph of $y = 1^x$ is the horizontal line $y = 1$.

Otherwise, the graph of $y = b^x$ has:
- $y$-intercept but no $x$-intercept,
- it goes to $\infty$ in one direction,
- it has the horizontal asymptote $y = 0$ in the other.

For $b > 1$, the graph of $b^x$ is like the graph of $2^x$ as below.

For $0 < b < 1$, the graph is like $\left(\frac{1}{2}\right)^x$.

Definition. $e^x$ is the unique exponential function $b^x$ whose the tangent at $(0,1)$ has slope 1. Fact: $e \approx 2.7$.

Definition. $\log_b(x)$, the log of $x$ to the base $b$, is the inverse of the exponential function $b^x$.

$\ln(x)$, the natural logarithm, $= \log_e(x)$ = the inverse of $e^x$.

Inverses act in opposite directions and inverses cancel.:

$y = \log_b(x)$ iff $b^y = x$.

$y = \ln(x)$ iff $e^y = x$.

$\log_b(b^y) = y$, $\log_b(1) = 0$, $\ln(e^x) = x$.

If the exponent base is $b$ instead of $e$, $\ln$ and $b^x$ don’t completely cancel; the bottom left equation becomes:

$\ln(b^x) = x \ln(b)$.

The exponent comes down to the outside.

Fact. $e^0 = 1 \Rightarrow 0 = \ln(1)$.

Know area, circumference, volume formulas for triangles, rectangles, boxes and cans. See inside front cover of the text.