Rectangular coordinates

\((a,b)\) is the point on the plane whose x-coordinate is \(a\) and y-coordinate is \(b\).

**Warning.** \((a,b)\) can mean a point or an interval.

**Definition.** The graph of an equation is the set of all points which satisfy the equation.

The graph of \(y=0\) is the x-axis.
The graph of \(x=0\) is the y-axis.

- Does \((5,6)\) lie on \(y = x^2 + 1\)?
  \((x,y) = (5,6)\) lies on \(y = x^2 + 1\) iff \(6 = (5)^2 + 1\) iff \(6 = 26\) iff no.

**Distance formula.** The distance \(d\) between points \((x_1, y_1)\) and \((x_2, y_2)\) is \(d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\).

- The distance between \((1,3)\) and \((2,5)\) is \(\sqrt{(1-2)^2 + (3-5)^2} = \sqrt{1^2 + 2^2} = \sqrt{5}\).

**Circles and completing the square**

**Circle Equation.** The circle with radius \(r\) and center \((a,b)\) has equation: \((x-a)^2 + (y-b)^2 = r^2\).

**Definition.** The graph of \(y = x^2 + 4\) is the parabola which opens up and crosses the y-axis at 4.

**Proof.** 
- If \((x,y)\) is on this circle
  \(x = \sqrt{\sqrt{2}}\) iff the distance between \((x,y)\) and \((a,b)\) is \(r\).

- Find the equation for the circle of radius 3 around \((1,2)\): \((x-1)^2 + (y-2)^2 = 3^2\).

**Facts.** For positive \(a, a = (\sqrt{a})^2\). For all \(a, \sqrt{a^2} = |a|\).

- Find the center and radius of the circle with equation: \((x+1)^2 + y^2 = 8\)
  \((x-(1))^2 + (y-0)^2 = (\sqrt{8})^2 = (\sqrt{2 \cdot 4})^2 = (2\sqrt{2})^2\).

- Completing the square. To make \(x^2 + ax\) a perfect square, add \((\frac{a}{2})^2\). \(x^2 + ax + (\frac{a}{2})^2 = (x + \frac{a}{2})^2\) is the latter is a “perfect” square.

- Find the center and radius of the circle with equation: \(4x^2 + 4y^2 + 4y - 79 = 0\)
  \(x^2 + y^2 + y - \frac{79}{4} = 0\)
  \(x^2 + y^2 + y = \frac{79}{4}\)
  \((x-0)^2 + (y^2 + y + \frac{1}{4}) = \frac{79}{4} + \frac{1}{4} = \frac{80}{4} = 20\)
  \((x-0)^2 + (y + \frac{1}{2})^2 = (\sqrt{20})^2 = (\sqrt{4 \cdot 5})^2\)
  \((x-0)^2 + (y - (-\frac{1}{2}))^2 = (2\sqrt{5})^2\).

- Center \((0, -\frac{1}{2})\), radius \(2\sqrt{5}\).

**Definition.** An \(x\)-intercept is the x-coordinate of a point where the graph crosses the x-axis. A \(y\)-intercept is the y-coordinate of a point where the graph crosses the y-axis. In fig. 2, \(x\) is the y-intercept; \(1, 5\) are the \(x\)-intercepts.

The graph crosses the x-axis when the y-coordinate is 0.
To find the \(x\)-intercepts, set \(y\) to 0 and solve for \(x\).
To find the \(y\)-intercepts, set \(x\) to 0 and solve for \(y\).

- Find the \(x\) and \(y\)-intercepts.
  \(y = x^2 - 4 = 0\)
  \(x\)-intercepts (set \(y\)=0): \(-2, 2\).
  \(y\)-intercepts (set \(x\)=0):
    \(-2^2 - 4 = 0, \quad y = -4\)
    \(y\)-intercept: \(-4\).

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  \(y = x^2 - 4 = 0\)
  \(x\)-intercepts:
    \(-2^2 - 4 = 0, \quad x^2 = -4, \quad x = -2, 2\)
  \(y\)-intercepts (set \(x\)=0):
    \(y = 0^2 - 4 = 0, \quad y = -4\)
    \(y\)-intercept: \(-4\).

**Straight lines: slopes and equations**

**Definition.** The slope of a line is the ratio of the vertical (height) change in \(y\) over a horizontal change in \(x\).

**Theorem.**
- The slope of the line through \((x_1, y_1)\) and \((x_2, y_2)\): \(m = \frac{y_2 - y_1}{x_2 - x_1}\).
- The equation of the line through \((x_1, y_1)\) with slope \(m\):
  \(y - y_1 = m(x - x_1)\)
- The equation of the line with slope \(m\) and \(y\)-intercept \(b\):
  \(y = mx + b\)
- The slope of a horizontal line is 0.
- The slope of a vertical line is undefined.
- Equation for the horizontal line through \((a, b)\): \(y = b\).
- Equation of the vertical line through \((a, b)\): \(x = a\).

**Line equation format.** In homework and tests, line equations must be in one of these four forms:
\(y = mx + b\), \(y = mx\), \(y = b\), \(x = a\).

- Find the line through \((2, 5)\) and \((4, 1)\). Check your answer.
  \(m = \frac{(5-1)/(2-4) = -2, \quad y-5 = -2(x-2), \quad y = -2x + 9\).
- Eq. of line with slope -8, \(y\)-intercept -6. \(y = -8x - 6\).